Available online at www.sciencedirect.com
ScienceDirect

The Journal of China Universities of Posts and Telecommunications

October 2016, 23(5): 1–9
www.sciencedirect.com/science/journal/10058885

http://jcupt.bupt.edu.cn

Performance analyses of OFDM AF relaying system in the presence of phase noise with APS and IPS

Huang Jing[1], Li Juhu[1] (✉), Zhang Haiyan[1]

School of Information, Beijing Forestry University, Beijing 100083, China

Abstract

This article investigates the significant performances of orthogonal frequency division multiplexing (OFDM)-based dual-hop system in the presence of phase noise (PN). A scenario with Rayleigh fading statistics on both hops is assumed. Amplification factor for this amplify-and-forward (AF) relay networks system is divided into two conditions, average power scaling (APS) and instantaneous power scaling (IPS). Before deriving signal-to-noise ratios (SNR) under APS and IPS, the Gaussianity of intercarrier interference (ICI) is proved firstly. The accurate closed-form expressions of end-to-end SNR cumulative distribution functions (CDF) and probability density functions (PDF) for both cases are obtained later. With the help of moment generating functions (MGF), we have closed-form asymptotic expressions of bit error rate (BER), which show that the BER of system in the presence of PN cannot exceed a fixed level even when SNR in high regime. Finally, simulations verify accuracy of the results. Conclusion analysis will provide a useful help in future application of the system.

Keywords OFDM-AF system, phase noise, average power scaling, instantaneous power scaling

1 Introduction

The demand on coverage area and capacity of wireless communication network have always been increasing due to fast developing technology introducing fancy wireless-equipped devices into our daily life. This drives the researches to put more effort to the representative model, dual-hop relay system. The relay terminal has two main relaying methods: AF and decode-and-forward (DF). In this study, we focus on AF relay systems due to the relatively small processing delay caused by decoding at the relay node compared with DF is shown in Ref. [1]. The available literatures on AF relaying schemes assume two different power constraints, APS and IPS. Depending on the capability of relay to estimate channel, these constraints scale the output power of the relayed signal in two different ways [2]. Some literatures use the two power constraints to analysis ideal condition. Other literatures study the system under situations which are not ideal, considering the complex of analysis, they use neither of power constraints situations but a coefficient to instead power constraint for performance analysis [3]. Although the conclusions can be applied for APS and IPS, the accuracy is not high. Thus we divide power constraint factors into the two specific situations under .

The most challenging problems in wireless communication is known as channel frequency selective fading and ICI. An excellent way to overcome these problems is using OFDM in multicarrier relaying system. Since Ref. [4] combined OFDM-based system with AF scheme, many literatures have discussed on performances of the method. In Ref. [5], Kocan et al. analyzed BER performance of dual-hop OFDM AF relay system with fixed gain (FG) at relay, implementing ordered subcarrier mapping (SCM) at relay. A scenario with Rayleigh fading statistics on both hops is assumed. In Ref. [6], Shah et al. presented the analysis of outage probability and derived accurate closed-form outage probability expressions for OFDM relay system while considering a fixed gain AF

Received date: 23-05-2016
Corresponding author: Li Juhu, E-mail: lijuhu@bjfu.edu.cn
DOI: 10.1016/S1005-8885(16)60051-1

relay system. Outage probability is easily obtained from PDF and CDF of SNR, considering space limitations we do not derivate and analysis it. In Ref. [7], Soliman et al. evaluated statistical performance for two-hop OFDM relay link with AF and derived closed-form expressions for the PDF of the SNR. The above references all assume in ideal channel, which surely did not exist in reality, but the research methods can be referenced in studying imperfect systems.

For non-ideal case, in Ref. [8], Rajkumar et al. take narrowband interference (NBI) into consideration, analyzed average BER performance of the OFDM based full duplex cognitive radio (CR) relay network. Other important interferences like PN, will destroy the orthogonality among subcarriers, introduce ICI. In practical environment we observe that a small phase drift at input of the local oscillator at each node can significantly limit the overall system BER. Also, there are some papers studying the influence of PN under other communication system models which prove the same conclusion above, but there is little investigation for OFDM AF system so far. Although in Refl. [9], Rabiei et al. took PN into account in OFDM AF relaying system, characterized the performance of the system, still it has limitations such as the system does not merge power constraints and its performance analysis only involved outage probability.

According to the derived PDF and CDF of SNR, we devise two methods to calculate BER. First is combing BER representations under various modulation cases with Gaussian function, which we have used to calculate BER for APS but find it is difficult under IPS. Second is using MGF. In Ref. [10], Kocan et al. analyzed BER performance with MGF of differential phase shift keying (DPSK) modulated dual-hop OFDM based relay systems implementing ordered SCM at relay station. This reference inspires us calculating BER with MGF for APS and IPS conditions.

This paper investigates the pivotal performances SNR and BER of OFDM AF system in the presence of PN. Firstly we prove that ICI is Gaussian random variable and investigate SNR of this system. Then end-to-end SNR CDF and PDF with APS and IPS of OFDM AF system with PN in Rayleigh fading channel are analyzed respectively. BER expressions are derived later by using MGF, which show that BER of the system cannot exceed a fixed level even when SNR in high regime. The analytical BER expressions are verified by numerical simulations.

The rest of the paper is organized as follows. In Sect. 2 the system model is established and the characters of ICI are analyzed in Sect. 3. SNR and end-to-end SNR CDF and PDF expressions are derived in Sect. 4. The performance analysis of BER in the presence of PN is presented in Sect. 5. Numerical results and discussion are given in Sect. 6. Sect. 7 concludes the article.

2 System model

In this section, we build the system model with PN firstly and then the PN model is presented.

2.1 System model with PN

Considering the OFDM dual-hop system with N subcarriers as shown in Fig. 1, constituted by source terminal S, half-duplex relay terminal R and destination terminal D. The signals at S are sent to R after OFDM modulation during first hop. Transmitting vector $\boldsymbol{X}=[X_0,\ldots,X_{N-1}]^{\mathrm{T}}$ is independent on all subcarriers. Influenced by PN, received signals at R in frequency domain can be written as:

$$\boldsymbol{Y}_{\mathrm{R}}=\boldsymbol{A}_{\mathrm{R}}\otimes(\boldsymbol{H}_{\mathrm{SR}}\sqrt{E_{\mathrm{S}}}\boldsymbol{X})+\boldsymbol{N}_{\mathrm{R}} \tag{1}$$

where $\boldsymbol{A}_{\mathrm{R}}$ is PN received at R, E_{S} is average signal energy received at R, $\boldsymbol{H}_{\mathrm{SR}}$ is channel frequency response matrix between S and R, $\boldsymbol{N}_{\mathrm{R}}$ is additive white Gaussian noise (AWGN), $\otimes$ denotes the convolution operator.

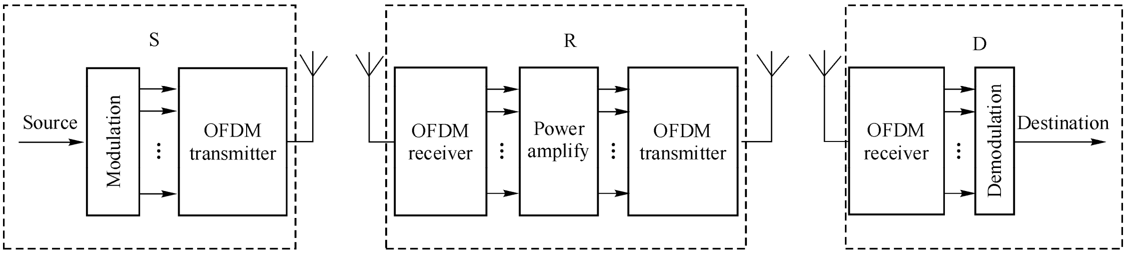

Fig. 1 Block diagram of OFDM AF relaying system with subcarrier mapping

The form of the ith subcarrier signal received at R can be derived as:

$$Y_{\mathrm{R},i} = A_{\mathrm{R},0}\sqrt{E_{\mathrm{S}}}H_{\mathrm{SR},i}X_i + I_{\mathrm{R},i} + N'_{\mathrm{R}} \tag{2}$$

where

$$I_{\mathrm{R},i} = \sum_{l=1}^{N-1} A_{\mathrm{R},l}\sqrt{E_{\mathrm{S}}}H_{\mathrm{SR},i-l}X_{i-l} \tag{3}$$

Obviously $I_{\mathrm{R},i}$ is ICI at relay node. $A_{\mathrm{R},0}$ is common phase error (CPE) in the ith subcarrier. $A_{\mathrm{R},l}$ is ICI caused by PN. $\boldsymbol{N}_{\mathrm{R}}$ is sampled with $N'_{\mathrm{R}} \sim \mathcal{CN}(0, N_0)$. We set $\mathrm{E}\{|X_i|^2\} = 1$, where $\mathrm{E}\{\bullet\}$ denotes the expectation operator.

Signals at relay will be amplified and transmitted to destination during the second hop. Assuming amplification factor is G, the signal received at destination can be expressed as:

$$\boldsymbol{Y}_{\mathrm{D}} = \boldsymbol{A}_{\mathrm{D}} \otimes (\boldsymbol{H}_{\mathrm{RD}} G\sqrt{E_{\mathrm{R}}}\boldsymbol{Y}_{\mathrm{R}}) + \boldsymbol{N}_{\mathrm{D}} \tag{4}$$

where $\boldsymbol{A}_{\mathrm{D}}$ is PN received at destination, $\boldsymbol{H}_{\mathrm{RD}}$ is channel frequency response matrix between R and D, $\mathrm{E}\{\|\boldsymbol{Y}_{\mathrm{R}}\|^2\} = 1$, $\boldsymbol{N}_{\mathrm{D}}$ is AWGN, samples with $N'_{\mathrm{D}} \sim \mathcal{CN}(0, N_0)$, E_{R} denote the average energy at R.

Suppose relay station maps the ith subcarrier of the first hop to kth subcarrier of the second hop, the channels frequency responses can be written as $H_{ab,k}$, $a,b \in \{\mathrm{S,R,D}\}$, $k = (0,\dots,N-1)$, which are independent on all subcarriers with average subcarrier symbol power $\mathrm{E}\{|H_{ab,k}|^2\} = \sigma^2_{H_{ab}}$, therefore the post-DFT signal on the kth subcarrier received at
D can be expressed as

$$\begin{aligned} Y_{\mathrm{D},k} &= GA_{\mathrm{D},0}H_{\mathrm{RD},k}\sqrt{E_{\mathrm{R}}}A_{\mathrm{R},0}\sqrt{E_{\mathrm{S}}}H_{\mathrm{SR},i}X_i + \\ &\quad GA_{\mathrm{D},0}H_{\mathrm{RD},k}\sqrt{E_{\mathrm{R}}}N'_{\mathrm{R}} + I_{1,k} + I_{2,k} + I_{3,k} + N'_{\mathrm{D}} \end{aligned} \tag{5}$$

where

$$\left.\begin{aligned} I_{1,k} &= G\sum_{l=1}^{N-1} A_{\mathrm{D},l}H_{\mathrm{RD},k-l}\sqrt{E_{\mathrm{R}}}A_{\mathrm{R},0}\sqrt{E_{\mathrm{s}}}H_{\mathrm{SR},k}X_k \\ I_{2,k} &= G\sum_{l=1}^{N-1} A_{\mathrm{D},l}H_{\mathrm{RD},k-l}\sqrt{E_{\mathrm{R}}}I_{\mathrm{R},k} \\ I_{3,k} &= G\sum_{l=1}^{N-1} A_{\mathrm{D},l}H_{\mathrm{RD},k-l}\sqrt{E_{\mathrm{R}}}N'_{\mathrm{R}} \end{aligned}\right\} \tag{6}$$

2.2 Phase noise model

We choose a commonly used PN model, Wiener model, whose accuracy is verified in many literatures. The existence of PN on relay and destination will mainly bring CPE and ICI, which will lead to the attenuation and rotation of desired signals. PN at receiver's (i.e., relay or destination) local oscillator introduces a random phase rotation of $\mathrm{e}^{\mathrm{j}\theta_k(n)}$ to the received signal in the time domain. PN at sample instant n is given by $\theta_k(n) = \theta_k(n+1) + \varepsilon$, where ε is a Gaussian random variable with zero mean and variance $2\pi\beta T_{\mathrm{s}}$, T_{s} is sampling interval and β is 3 dB PN bandwidth, $n = 1,\dots,N-1$ with N is the number of subcarriers.

PN in frequency-domain is presented by

$$A_{t,l} = \frac{1}{N}\sum_{i=0}^{N-1} \mathrm{e}^{\mathrm{j}\theta_{N_{\mathrm{g}}+i}}\,\mathrm{e}^{-\mathrm{j}\frac{2\pi li}{N}};\quad t \in \{\mathrm{R,D}\} \tag{7}$$

where N_{g} is the length of guard interval (GI). Combine the symbols used in the formulas before and Ref. [11], we can get PN variance expression:

$$\mathrm{E}\{|A_{t,l}|^2\} = \frac{1}{N^2}\left[\frac{\rho_l^{N+1} - (N+1)\rho_l + N}{(\rho_l - 1)^2} - N\right] \tag{8}$$

where $\rho_l = \exp[(2\pi\mathrm{j}l/N) - (\pi\beta_t T_{\mathrm{s}}/N)]$. By substituting $l = 0$ in Eq. (8), variance of the CPE is obtained. If the variance of PN is infinitesimal (i.e., $\sigma_\theta^2 \approx 0$) which is the case in practice, variance of the PN term can be simplified by approximating the exponential term with Taylor series and represented as

$$\left.\begin{aligned} &\mathrm{E}\{|A_{t,0}|^2\} = 1 - \frac{\pi\beta_t T_{\mathrm{s}}}{3} = C_{t,0} \\ &\mathrm{E}\left\{\sum_{l=1}^{N-1}|A_{t,l}|^2\right\} = 1 - \mathrm{E}\{|A_{t,0}|^2\} = 1 - C_{t,0} \end{aligned}\right\} \tag{9}$$

We can see that when PN does not exist, $C_{t,0}$ is equal to 1.

3 Statistical analysis of intercarrier interference

In this section we calculate the mean and variance of ICI and prove its Gaussian. Assume the transmitted data $\boldsymbol{X} = [X_0,\dots,X_{N-1}]^{\mathrm{T}}$ on all subcarriers are independent as well as channels frequency response $H_{ab,k}$, $a,b \in \{\mathrm{S,R,D}\}$, $(k = 0,\dots,N-1)$.

3.1 Mean and variance of ICI

Considering mary quadrature amplitude modulation (M-QAM) modulation in our system, X_k and $I_{\mathrm{R},k}$ are zero mean. $H_{ab,k}$ and X_k are independent of each other. Use the hypothesis above, the variance of $I_{\mathrm{R},k}$ can be expressed

$$\sigma^2_{\mathrm{ICI}_{\mathrm{R},k}} = \mathrm{E}\left\{\left|I_{\mathrm{R},k}\right|^2\right\} = \mathrm{E}\left\{\left|\sum_{l=1}^{N-1} A_{\mathrm{R},l}\sqrt{E_{\mathrm{SR}}}H_{\mathrm{SR},k-l}X_{k-l}\right|^2\right\} = E_{\mathrm{S}}\sigma^2_{H_{\mathrm{SR}}}\left(1-C_{\mathrm{SR}}\right) \tag{10}$$

$I_{2,k}$, $I_{2,k}$ and $I_{3,k}$ are also zero mean. Similarly their variances can be given as

$$\left.\begin{aligned}
\sigma^2_{I_{1,k}} &= \mathrm{E}\left\{\left|I_{1,k}\right|^2\right\} = G^2E_{\mathrm{S}}S_{\mathrm{SR}}\left(1-C_{\mathrm{RD}}\right)\sigma^2_{H_{\mathrm{SR}}}\sigma^2_{H_{\mathrm{RD}}}\\
\sigma^2_{I_{2,k}} &= \mathrm{E}\left\{\left|I_{2,k}\right|^2\right\} = G^2E_{\mathrm{S}}\left(1-C_{\mathrm{SR}}\right)\sigma^2_{H_{\mathrm{SR}}}\sigma^2_{H_{\mathrm{RD}}}\\
\sigma^2_{I_{3,k}} &= \mathrm{E}\left\{\left|I_{3,k}\right|^2\right\} = G^2\left(1-C_{\mathrm{RD}}\right)\sigma^2_{H_{\mathrm{RD}}}N_0
\end{aligned}\right\} \tag{11}$$

3.2 Gaussianity of ICI

Although the ICI term $I_{\mathrm{R},k}$ is modeled as Gaussian random variable with zero mean and variance $\sigma^2_{I_{\mathrm{R}}}$ extensively in literatures, to the best of authors' knowledge, no proof is given so far to validate this assumption. Here we give a simple proof to the asymptotic Gaussianity of the ICI. To prove the Gaussianity of ICI, we use Lyapunov's central limit theorem [12], which is restated here in a convenient form.

Lemma 1 Lyapunov. If $\Lambda_1,\Lambda_2,\ldots,\Lambda_N$ are independent random variables each with mean μ_i variance σ_i^2 and finite absolute third moment η_i^3. And if $\lim\limits_{N\to\infty}(\eta/\sigma)=0$ where $\eta=\left(\sum\limits_{i=0}^{N-1}\eta_i^3\right)^{1/3}$, $\sigma=\left(\sum\limits_{i=0}^{N-1}\sigma_i^2\right)^{1/2}$. Then $\sum\limits_{i=0}^{N-1}\Lambda_i$ is asymptotically Gaussian.

Proof Gaussianity of ICI. We rewrite the ICI term in Eq. (3) as $I_{\mathrm{R},k}=\sum\limits_{r=1}^{N-1}\Lambda_r$, where $\Lambda_r = A_{\mathrm{R},l}\sqrt{E_{\mathrm{SR}}}H_{\mathrm{SR},k-l}X_{k-l}$. We need to evaluate the mean, variance and third absolute moment of Λ_r in order to proceed with the proof. Since the data symbols are independent of the channels and the channels are a zero mean complex Gaussian random variable, mean μ_{Λ_r}, variance $\sigma^2_{\Lambda_r}$ and finite absolute third moment $\eta^3_{\Lambda_r}$ can be calculated as

$$\left.\begin{aligned}
\mu_{\Lambda_r} &= \mathrm{E}\left\{A_{\mathrm{R},l}\sqrt{E_{\mathrm{SR}}}H_{\mathrm{SR},k-l}X_{k-l}\right\}=0\\
\sigma^2_{\Lambda_r} &= \mathrm{E}\left\{\left|A_{\mathrm{R},l}\sqrt{E_{\mathrm{SR}}}H_{\mathrm{SR},k-l}X_{k-l}\right|^2\right\} = E_{\mathrm{S}}\sigma^2_{H_{\mathrm{SR}}}\left|C_{\mathrm{SR}}\right|^2\\
\eta^3_{\Lambda_r} &= \mathrm{E}\left\{\left|A_{\mathrm{R},l}\sqrt{E_{\mathrm{SR}}}H_{\mathrm{SR},k-l}X_{k-l}\right|^3\right\} = \left(E_{\mathrm{S}}\sigma^2_{H_{\mathrm{SR}}}\right)^{\frac{3}{2}}\left|C_{\mathrm{SR}}\right|^3
\end{aligned}\right\} \tag{12}$$

From Eq. (12), we can get the following expression

$$\lim_{N\to\infty}\frac{\left(\sum\limits_{r=1}^{N-1}\left(E_{\mathrm{S}}\sigma^2_{H_{\mathrm{SR}}}\right)^{\frac{3}{2}}\left|C_{\mathrm{SR}}\right|^3\right)^{\frac{1}{3}}}{\left(\sum\limits_{r=1}^{N-1}E_{\mathrm{S}}\sigma^2_{H_{\mathrm{SR}}}\left|C_{\mathrm{SR}}\right|^2\right)^{\frac{1}{2}}} \leqslant \lim_{N\to\infty}\frac{\left(\max\left\{\left|C_{\mathrm{SR}}\right|^3\right\}\right)^{\frac{1}{3}}}{(N-1)^{\frac{1}{6}}\left(\min\left\{\left|C_{\mathrm{SR}}\right|^2\right\}\right)^{\frac{1}{2}}} = 0 \tag{13}$$

According to what has been discussed above, we can easily conclude that $I_{\mathrm{R},k}$ is Gaussian random variable. Similarly, $I_{1,k}$, $I_{2,k}$ and $I_{3,k}$ are Gaussian random variables.

4 The end-to-end SNR CDF and PDF

In this section we present the end-to-end SNR CDF and PDF with APS and IPS constraints. These results will be useful for deriving expressions for the theoretical BER.

4.1 APS

The power scaling factor is $G^2 = 1/\left(E_{\mathrm{S}}+N_0\right)$ for APS. Rewrite Eq. (5) as

$$Y_{\mathrm{D},k} = \sqrt{\lambda E_{\mathrm{R}}}A_{\mathrm{D},0}H_{\mathrm{RD},k}A_{\mathrm{R},0}H_{\mathrm{SR},k}X_k + N_0 \tag{14}$$

where $\lambda_{\mathrm{APS}} = E_{\mathrm{S}}N_0\left(E_{\mathrm{R}}A^2_{\mathrm{D},0}H^2_{\mathrm{RD},k}N_0 + \left(E_{\mathrm{S}}+N_0\right)\left(\sigma^2_{I_{1,k}}+\sigma^2_{I_{3,k}}+\sigma^2_{I_{2,k}}\right)+\left(E_{\mathrm{S}}+N_0\right)N_0\right)^{-1}$

S-R and R-D Rayleigh fading channels are independent among subcarriers, so PDF and CDF of SNR in each subchannel can be written as $f_{ab}(x)=\bar{\gamma}_{ab}^{-1}\exp\left(-\bar{\gamma}_{ab}^{-1}x\right)$ and $F_{ab}(x)=1-\exp\left(-\left(1/\bar{\gamma}_{ab}\right)x\right)$, $a,b\in\{\mathrm{S,R,D}\}$ and as Ref. [13].

Simplifying Eq. (14) to use the form of expressions above, we get the end-to-end SNR on kth subcarrier for APS as

$$\gamma_{\text{end}}^{\text{APS}}(k)=\frac{C_{\text{SR}}C_{\text{RD}}\gamma_{\text{SR}}(i)\gamma_{\text{RD}}(k)}{C_{\text{RD}}\gamma_{\text{RD}}(k)+\rho_{\text{APS}}} \tag{15}$$

where $\rho_{\text{APS}}=\left[(1-C_{\text{SR}})+C_{\text{SR}}(1-C_{\text{RD}})\right]\overline{\gamma}_{\text{SR}}\overline{\gamma}_{\text{RD}}+(1-C_{\text{RD}})\cdot\overline{\gamma}_{\text{RD}}+(E_{\text{S}}+N_0)/N_0$, $\gamma_{\text{SR}}(i)$ and $\gamma_{\text{RD}}(k)$ denote instantaneous SNR on ith subcarrier of first hop and kth subcarrier of second hop respectively. So the end-to-end CDF of SNR can be expressed as

$$F_{\gamma_{\text{end}}^{\text{APS}}}(x)=\int_0^{\infty}P\left(\frac{C_{\text{SR}}C_{\text{RD}}\gamma_{\text{SR}}(i)\gamma_{\text{RD}}(k)}{C_{\text{RD}}\gamma_{\text{RD}}(k)+\rho_{\text{APS}}}<x\,\middle|\,\gamma_{\text{RD}}(k)\right)\cdot f_{\text{RD}}\left(\gamma_{\text{RD}}(k)\right)\mathrm{d}\gamma_{\text{RD}}(k) \tag{16}$$

Integrating Eq. (16) can reach close-form CDF expression of SNR for APS as Eq. (17), based on the analysis of Appendix A.

$$F_{\gamma_{\text{end}}^{\text{APS}}}(x)=1-\mathrm{e}^{-\frac{x}{\overline{\gamma}_{\text{RD}}C_{\text{RD}}}}+\frac{1}{C_{\text{RD}}\overline{\gamma}_{\text{RD}}}\mathrm{e}^{-\frac{x}{\overline{\gamma}_{\text{RD}}C_{\text{RD}}}}\left[\overline{\gamma}_{\text{RD}}C_{\text{RD}}-2\sqrt{\frac{\rho_{\text{APS}}\overline{\gamma}_{\text{RD}}C_{\text{RD}}x}{C_{\text{SR}}\overline{\gamma}_{\text{SR}}}}\mathrm{K}_1\left(2\sqrt{\frac{\rho_{\text{APS}}x}{C_{\text{SR}}C_{\text{RD}}\overline{\gamma}_{\text{SR}}\overline{\gamma}_{\text{RD}}}}\right)\right] \tag{17}$$

And the end-to-end PDF of SNR through derivations shown in Appendix B is

$$f_{\gamma_{\text{end}}^{\text{APS}}}(x)=\frac{2}{C_{\text{SR}}\overline{\gamma}_{\text{SR}}}\mathrm{e}^{-\frac{x}{C_{\text{SR}}\overline{\gamma}_{\text{SR}}}}\left[\sqrt{\frac{\rho_{\text{APS}}x}{C_{\text{SR}}C_{\text{RD}}\overline{\gamma}_{\text{SR}}\overline{\gamma}_{\text{RD}}}}\cdot\mathrm{K}_1\left(2\sqrt{\frac{\rho_{\text{APS}}x}{C_{\text{SR}}C_{\text{RD}}\overline{\gamma}_{\text{SR}}\overline{\gamma}_{\text{RD}}}}\right)+\frac{\rho_{\text{APS}}}{C_{\text{SR}}C_{\text{RD}}\overline{\gamma}_{\text{RD}}}\cdot\mathrm{K}_0\left(2\sqrt{\frac{\rho_{\text{APS}}x}{C_{\text{SR}}C_{\text{RD}}\overline{\gamma}_{\text{SR}}\overline{\gamma}_{\text{RD}}}}\right)\right] \tag{18}$$

where $\mathrm{K}_1(\bullet)$ and $\mathrm{K}_0(\bullet)$ are first and zero order modified Bessel functions of second kind [14].

4.2 IPS

Under IPS constraint $G^2=1/\left(E_{\text{S}}|H_{\text{SR}}|^2+N_0\right)$, λ in Eq. (14) is

$$\lambda_{\text{IPS}}=E_{\text{S}}N_0\left(E_{\text{R}}A_{\text{D},0}^2H_{\text{RD},k}^2N_0+\left(E_{\text{S}}|H_{\text{SR}}|^2+N_0\right)\left(\sigma_{I_{1,k}}^2+\sigma_{I_{3,k}}^2+\sigma_{I_{2,k}}^2\right)+\left(E_{\text{S}}|H_{\text{SR}}|^2+N_0\right)N_0\right)^{-1}$$

Similarly as APS, the end-to-end SNR on kth subcarrier for IPS is shown as

$$\gamma_{\text{end}}^{\text{IPS}}(k)=\frac{C_{\text{SR}}C_{\text{RD}}\gamma_{\text{SR}}(i)\gamma_{\text{RD}}(k)}{C_{\text{RD}}\gamma_{\text{RD}}(k)+\gamma_{\text{SR}}(i)+\rho_{\text{IPS}}} \tag{19}$$

where $\rho_{\text{IPS}}=\left[(1-C_{\text{SR}})+C_{\text{SR}}(1-C_{\text{RD}})\right]\overline{\gamma}_{\text{SR}}\overline{\gamma}_{\text{RD}}+(1-C_{\text{RD}})\cdot\overline{\gamma}_{\text{RD}}+1$.

We report a lemma on CDF of independent exponent variates firstly for deriving the expression of theoretical SNR PDF under IPS.

Lemma 2 [15] Let X_1, X_2 be two independent exponent RVs with parameters β_1 and β_2, respectively [i.e. $X_i\sim\varepsilon(\beta_i)$, $i=1,2$]. Then, the CDF of $X=\left[(X_1X_2)/(X_1+X_2+b)\right]$, where b is a constant, is given by

$$F_{\text{X}}(x)=1-2\,\mathrm{e}^{-(\beta_1+\beta_2)x}\sqrt{\beta_1\beta_2x(x+b)}\mathrm{K}_1\left(2\sqrt{\beta_1\beta_2x(x+b)}\right) \tag{20}$$

Lemma 2 helps get the CDF of SNR expression:

$$F_{\gamma_{\text{end}}^{\text{IPS}}(k)}(x)=1-2\,\mathrm{e}^{-\left(\frac{1}{\overline{\gamma}_{\text{SR}}}+\frac{1}{\overline{\gamma}_{\text{RD}}C_{\text{RD}}}\right)\frac{x}{C_{\text{SR}}}}\sqrt{\frac{x}{C_{\text{SR}}C_{\text{RD}}\overline{\gamma}_{\text{SR}}\overline{\gamma}_{\text{RD}}}}\cdot\sqrt{\left(\frac{x}{C_{\text{SR}}}+\rho_{\text{IPS}}\right)}\mathrm{K}_1\left(2\sqrt{\frac{x}{C_{\text{SR}}C_{\text{RD}}\overline{\gamma}_{\text{SR}}\overline{\gamma}_{\text{RD}}}\left(\frac{x}{C_{\text{SR}}}+\rho_{\text{IPS}}\right)}\right) \tag{21}$$

To get the PDF of $\gamma_{\text{end}}^{\text{IPS}}(k)$, we differentiate Eq. (21) with the help of Bessel derivative equation in Ref. [16] and obtain the PDF as:

$$f_{\gamma_{\text{end}}^{\text{IPS}}(k)}(x)=\frac{2}{C_{\text{SR}}}\mathrm{e}^{-\left(\frac{1}{\overline{\gamma}_{\text{SR}}}+\frac{1}{\overline{\gamma}_{\text{RD}}C_{\text{RD}}}\right)\frac{x}{C_{\text{SR}}}}\left[\left(\frac{1}{\overline{\gamma}_{\text{SR}}}+\frac{1}{\overline{\gamma}_{\text{RD}}C_{\text{RD}}}\right)\cdot\sqrt{\frac{x}{C_{\text{SR}}C_{\text{RD}}\overline{\gamma}_{\text{SR}}\overline{\gamma}_{\text{RD}}}\left(\frac{x}{C_{\text{SR}}}+\rho_{\text{IPS}}\right)}\cdot\mathrm{K}_1\left(2\sqrt{\frac{x}{C_{\text{SR}}C_{\text{RD}}\overline{\gamma}_{\text{SR}}\overline{\gamma}_{\text{RD}}}\left(\frac{x}{C_{\text{SR}}}+\rho_{\text{IPS}}\right)}\right)+\frac{x}{C_{\text{SR}}C_{\text{RD}}\overline{\gamma}_{\text{RD}}}\left(\frac{2x}{C_{\text{SR}}}+\rho_{\text{IPS}}\right)\cdot\mathrm{K}_0\left(2\sqrt{\frac{x}{C_{\text{SR}}C_{\text{RD}}\overline{\gamma}_{\text{SR}}\overline{\gamma}_{\text{RD}}}\left(\frac{x}{C_{\text{SR}}}+\rho_{\text{IPS}}\right)}\right)\right] \tag{22}$$

5 Performance analysis

In this section, we derive closed-form expressions for the MGF with APS and IPS constraints and then BER can be calculated via the MGF-based approach.

5.1 MGF for APS and IPS

5.1.1 MGF for APS

Attempting to derive the exact MGF for APS with Eq. (18) is difficult. Accordance with the actual accident situation, we assume transmission powers in source node and relay node are relatively large, i.e. $\bar{\gamma}_{\text{SR}} \to \infty$ and $\bar{\gamma}_{\text{RD}} \to \infty$, and the carrier frequency offset is small, i.e. $C_{\text{SR}} \approx 1$, $C_{\text{RD}} \approx 1$. Then $k\sqrt{(1-C_{\text{SR}}C_{\text{RD}})x/(C_{\text{SR}}C_{\text{RD}})} \to 0$. Under these assumptions and equations in Ref. [15], the approximate expression of PDF is:

$$f_{\gamma_{\text{end}}^{\text{APS}}}(x) \approx \frac{1}{C_{\text{SR}}\bar{\gamma}_{\text{SR}}}\mathrm{e}^{-\frac{x}{C_{\text{SR}}\bar{\gamma}_{\text{RS}}}} + \frac{2}{C_{\text{SR}}\bar{\gamma}_{\text{SR}}}\mathrm{e}^{-\frac{x}{C_{\text{SR}}\bar{\gamma}_{\text{SR}}}}\frac{(1-C_{\text{SR}}C_{\text{RD}})}{C_{\text{SR}}C_{\text{RD}}\bar{\gamma}_{\text{SR}}}\cdot \ln\left(2\sqrt{\frac{(1-C_{\text{SR}}C_{\text{RD}})x}{C_{\text{SR}}C_{\text{RD}}}}\right) \tag{23}$$

To obtain system error rate, we chose a more concise method which based on the relationship between error rate and MGF. MGF of $\gamma_{\text{end}}^{\text{APS}}(k)$ is $M_{\gamma_{\text{end}}^{\text{APS}}(k)}(s) = \mathrm{E}\left\{\exp\left(-s\gamma_{\text{end}}^{\text{APS}}(k)\right)\right\}$, and it can be rewritten with the help of Eq. (23) as

$$M_{\gamma_{\text{end}}^{\text{APS}}(k)}(s) = \int_0^{\infty}\mathrm{e}^{-sx}f_{\gamma_{\text{end}}^{\text{APS}}(k)}(x)\mathrm{d}x = \frac{1}{1+sC_{\text{SR}}\bar{\gamma}_{\text{SR}}} + \frac{\dfrac{2(1-C_{\text{SR}}C_{\text{RD}})}{C_{\text{SR}}C_{\text{RD}}}}{\left(\dfrac{1}{C_{\text{SR}}\bar{\gamma}_{\text{SR}}}+s\right)^2}\left[2\ln\left(\frac{1}{C_{\text{SR}}\bar{\gamma}_{\text{SR}}}+s\right) - \left(\frac{4(1-C_{\text{SR}}C_{\text{RD}})}{C_{\text{SR}}C_{\text{RD}}}\right)+2\zeta-2\right] \tag{24}$$

where ζ is Euler's constant.

5.1.2 MGF for IPS

We derive BER for IPS with the same assumptions as APS. Therefore $\sqrt{\left(x/(C_{\text{SR}}C_{\text{RD}}\bar{\gamma}_{\text{SR}}\bar{\gamma}_{\text{RD}})\right)\left((x/C_{\text{SR}})+\rho_{\text{ISP}}\right)} \to 0$, $(x/C_{\text{SR}})^2 \gg x\rho_{\text{IPS}}/C_{\text{SR}}$, and the approximate expression of PDF is:

$$f_{\gamma_{\text{end}}^{\text{IPS}}(k)}(x) \approx \frac{1}{C_{\text{SR}}}\left(\frac{1}{\bar{\gamma}_{\text{SR}}}+\frac{1}{\bar{\gamma}_{\text{RD}}C_{\text{RD}}}\right)\mathrm{e}^{-\left(\frac{1}{\bar{\gamma}_{\text{SR}}}+\frac{1}{\bar{\gamma}_{\text{RD}}C_{\text{RD}}}\right)\frac{x}{C_{\text{SR}}}} - \left(\frac{2}{C_{\text{SR}}}\right)^2\frac{x}{C_{\text{SR}}C_{\text{RD}}\bar{\gamma}_{\text{RD}}}\mathrm{e}^{-\left(\frac{1}{\bar{\gamma}_{\text{SR}}}+\frac{1}{\bar{\gamma}_{\text{RD}}C_{\text{RD}}}\right)\frac{x}{C_{\text{SR}}}}\cdot \ln\left(\frac{2x}{C_{\text{SR}}}\sqrt{\frac{1}{\bar{\gamma}_{\text{SR}}\bar{\gamma}_{\text{RD}}C_{\text{RD}}}}\right) \tag{25}$$

MGF of $\gamma_{\text{end}}^{\text{APS}}(k)$ is $M_{\gamma_{\text{end}}^{\text{IPS}}(k)}(s) = \mathrm{E}\left\{\exp\left(-s\gamma_{\text{end}}^{\text{IPS}}(k)\right)\right\}$, and it can be rewritten as

$$M_{\gamma_{\text{end}}^{\text{ISP}}(k)}(s) = \int_0^{\infty}\mathrm{e}^{-sx}f_{\gamma_{\text{end}}^{\text{IPS}}(k)}(x)\mathrm{d}x = \frac{\dfrac{1}{C_{\text{SR}}}\left(\dfrac{1}{\bar{\gamma}_{\text{SR}}}+\dfrac{1}{\bar{\gamma}_{\text{RD}}C_{\text{RD}}}\right)}{\dfrac{1}{C_{\text{SR}}}\left(\dfrac{1}{\bar{\gamma}_{\text{SR}}}+\dfrac{1}{\bar{\gamma}_{\text{RD}}C_{\text{RD}}}\right)+s} + \frac{\dfrac{2}{\bar{\gamma}_{\text{SR}}\bar{\gamma}_{\text{RD}}(C_{\text{SR}})^2C_{\text{RD}}}}{\left(\dfrac{1}{C_{\text{SR}}}\left(\dfrac{1}{\bar{\gamma}_{\text{SR}}}+\dfrac{1}{\bar{\gamma}_{\text{RD}}C_{\text{RD}}}\right)+s\right)^2}\cdot\left[2\ln\left(\frac{1}{C_{\text{SR}}}\left(\frac{1}{\bar{\gamma}_{\text{SR}}}+\frac{1}{\bar{\gamma}_{\text{RD}}C_{\text{RD}}}\right)+s\right) - \left(\frac{4}{\bar{\gamma}_{\text{SR}}\bar{\gamma}_{\text{RD}}(C_{\text{SR}})^2C_{\text{RD}}}\right)+2\zeta-2\right] \tag{26}$$

5.2 BER

In Ref. [17], Su et al. revealed the relationship between error rate and MGF when the system is M-QAM modulations as

$$P_{\text{e}} \simeq \frac{4K}{\pi}\int_0^{\frac{\pi}{2}}M_{\gamma}\left(\frac{b_{\text{QAM}}}{2\sin^2\theta}\right)\mathrm{d}\theta - \frac{4K^2}{\pi}\int_0^{\frac{\pi}{4}}M_{\gamma}\left(\frac{b_{\text{QAM}}}{2\sin^2\theta}\right)\mathrm{d}\theta \tag{27}$$

where $K = 1-\left(1/\sqrt{M}\right)$, $b_{\text{QAM}} = 3/(M-1)$. Basing on Eq. (27), we can get the expression of error rate using MGF as Eqs. (24) and (26).

6 Numerical result

This section analyses the BER simulation results for the system constraints in Rayleigh fading channels. Although the derived expressions can compute the BER for any values of $\bar{\gamma}_{\text{SR}}$ and $\bar{\gamma}_{\text{RD}}$, we limit the discussion to the balance link case ($E_{\text{S}} = E_{\text{R}}, \sigma_{\text{R}}^2 = \sigma_{\text{D}}^2 = N_0, \bar{\gamma}_{\text{SR}} = \bar{\gamma}_{\text{RD}}$). The OFDM system has $N = 16$ subcarrier. Then the simulation figures are shown as follows.

In Figs. 2 and 3, investigate the BER performances under different PN as a function of SNR. 16QAM modulated OFDM AF relay system over APS and IPS is precondition. A fairly good agreement between theoretical and simulated results confirms the validity of the conclusion presented in this article. In Fig. 2, the error floor of BER is approximately 9.5×10^{-3} when PN linewidthes at relay and destination are 1 Hz whereas 3.8×10^{-2} when 10 Hz, compared with 9×10^{-3} when 1 Hz and 3.2×10^{-2} when 10 Hz in Fig. 3. Thus IPS has a lower BER than APS when PN for relay and destination are same as shown in Figs. 2 and 3.

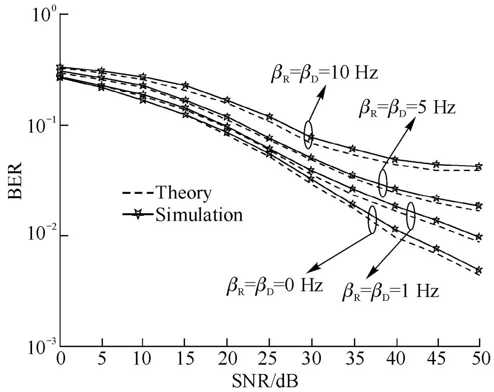

Fig. 2 The BER of OFDM AF system for APS

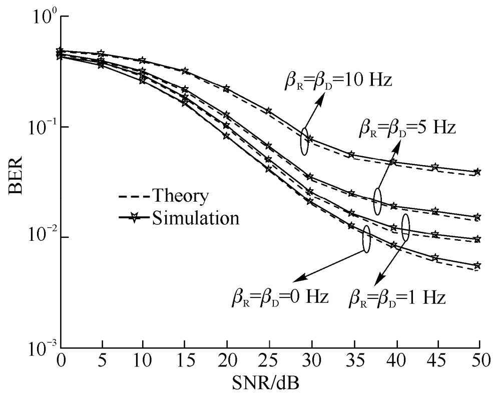

Fig. 3 The BER of OFDM AF system for IPS

From Figs. 2 and 3, we can conclude that:

1) Comparing with ideal system without PN, shown as the $\beta = 0$ Hz curve in Figs. 2 and 3, system with PN performance worse in BER. The larger PN is, worse the system performance in BER.

2) For the low SNR regime, the different effects of PN on BER performance of OFDM AF relay system with different amplification factors are not obvious. Since PN degrades BER performance by introducing phase offset as well as ICI to useful signal and destroys subcarrier orthogonally. Only little subcarrier power leaks to adjacent subcarrier under small transmitting power.

3) The OFDM system becomes more sensitive on PN in high region. For the high SNR regime with large PN, BER performances of OFDM AF relay system tend to error floor rather than zero. Because when PN is large, the power of subcarrier leaks a lot to the adjacent subcarriers.

4) Under different PN, IPS always performs better than APS in BER.

Figs. 4 and 5 give analytical and simulation results for 4QAM modulation for OFDM AF relay system implementing both APS and IPS, for different total numbers of subcarrier.

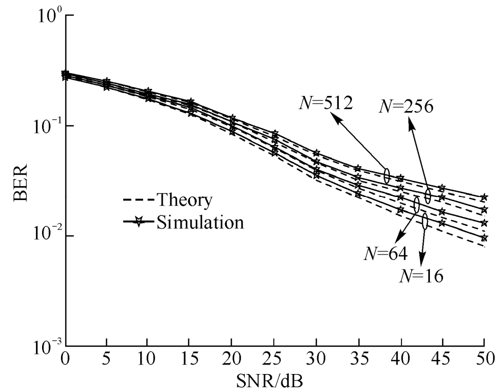

Fig. 4 The BER of OFDM AF system for APS for different numbers of subcarrier

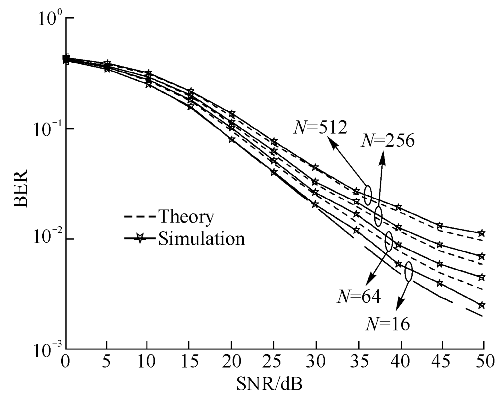

Fig. 5 The BER of OFDM AF system for IPS for different numbers of subcarrier

Figs. 4–5 depict that:

1) With the increasing of subcarrier number, the BER performance is descending. That because when the number of subcarrier increases, the frequency interval between subcarriers will be reduced, and frequency offset caused by Doppler channel expansion will lead to the increase of ICI, therefore the subcarrier will be more susceptible to ICI damages.

2) Comparing Fig. 4 and Fig. 5, it can be seen that the BER performances for IPS conditions is more sensitive to the numbers of subcarrier. Under the case of APS only

average signal energy received at R is considered. But under IPS, the channel frequency response matrix, which is closely related to the number of subcarrier, is also taken into consideration. Apparently the phenomenon above verifies the correctness of the theoretical conclusions of this paper.

7 Conclusions

In this paper, we have examined the performance of OFDM AF system with APS and IPS over exponential correlated Rayleigh fading channels and uncorrelated Rician fading channels in presence of phase noise. A direct expression for post-processing SNR is derived with phase noise. Based on the MGF, we derived closed-form BER of system for both APS and IPS conditions. The simulation results demonstrated that a small phase noise has great impact on the BER performance.

Acknowledgements

This work was supported by the Fundamental Research Funds for the Central Universities of China (TD2014-01).

Appendix A Proof of end-to-end SNR CDF expressions

Eq. (16) can be rewriten as the sum of two integrals as

$$F_{\gamma_{\text{end}}^{\text{APS}}}(x)=\int_0^x P\left(\frac{C_{\text{SR}}C_{\text{RD}}\gamma_{\text{SR}}(i)\gamma_{\text{RD}}(k)}{C_{\text{RD}}\gamma_{\text{RD}}(k)+\rho_{\text{ASP}}}<x\,\middle|\,\gamma_{\text{RD}}(k)\right)\cdot f_{\text{RD}}(\gamma_{\text{RD}}(k))\,\mathrm{d}\gamma_{\text{RD}}(k)+\int_x^\infty P\left(\frac{C_{\text{SR}}C_{\text{RD}}\gamma_{\text{SR}}(i)\gamma_{\text{RD}}(k)}{C_{\text{RD}}\gamma_{\text{RD}}(k)+\rho_{\text{ASP}}}<x\,\middle|\,\gamma_{\text{RD}}(k)\right)\cdot f_{\text{RD}}(\gamma_{\text{RD}}(k))\,\mathrm{d}\gamma_{\text{RD}}(k) \quad \text{(A.1)}$$

It should be noted that the first integral trem $P\left(C_{\text{SR}}\gamma_{\text{SR}}(i)\left(C_{\text{RD}}\gamma_{\text{RD}}(k)-\gamma_{\text{th}}\right)\leqslant\rho_{\text{APS}}x\right)$ equals to unity for $0\leqslant\gamma_{\text{SR}}(i)\leqslant x$. Since $\mathrm{d}F_X(x)/\mathrm{d}x=1-F_X(x)$ where $F_X(x)=\int_{-\infty}^x f_X(x)\mathrm{d}x$, Eq. (A.1) can be simplified as

$$F_{\gamma_{\text{end}}^{\text{APS}}}(x)=1-\int_0^\infty\frac{1}{C_{\text{RD}}}f_{\text{RD}}\left(\frac{y+x}{C_{\text{RD}}}\right)\mathrm{d}y+\int_0^\infty\frac{1}{C_{\text{RD}}}F_{\text{SR}}\left(\frac{\rho_{\text{APS}}x}{C_{\text{SR}}y}\right)\cdot f_{\text{RD}}\left(\frac{y+x}{C_{\text{RD}}}\right)\mathrm{d}y \quad \text{(A.2)}$$

The first integral in Eq. (A.2) can be written as Eq. (A.3) using $f_{\text{RD}}(x)=(1/\overline{\gamma}_{\text{RD}})\exp\left(-(1/\overline{\gamma}_{\text{RD}})x\right)$,

$$\int_0^\infty\frac{1}{C_{\text{RD}}}f_{\text{RD}}\left(\frac{y+x}{C_{\text{RD}}}\right)\mathrm{d}y=\mathrm{e}^{-\frac{x}{\overline{\gamma}_{\text{RD}}C_{\text{RD}}}} \quad \text{(A.3)}$$

The CDF $F_{\gamma_{\text{end}}^{\text{APS}}}(x)$ of the SNR in each S-R subchannel can be derived by integral of the PDF function with the variable $\rho_{\text{APS}}x/(C_{\text{SR}}y)$. Then under the help of Ref. [16] the second integral of $F_{\gamma_{\text{end}}^{\text{APS}}}(x)$ in Eq. (A.2) can be rewritten as Eq. (A.4).

$$\int_0^\infty\frac{1}{C_{\text{RD}}}F_{\text{SR}}\left(\frac{\rho_{\text{APS}}x}{C_{\text{SR}}y}\right)f_{\text{RD}}\left(\frac{y+x}{C_{\text{RD}}}\right)\mathrm{d}y=\frac{1}{\overline{\gamma}_{\text{RD}}C_{\text{RD}}}\mathrm{e}^{-\frac{x}{\overline{\gamma}_{\text{RD}}C_{\text{RD}}}}\cdot\left[\overline{\gamma}_{\text{RD}}C_{\text{RD}}-2\sqrt{\frac{\rho_{\text{APS}}\overline{\gamma}_{\text{RD}}C_{\text{RD}}x}{C_{\text{SR}}\overline{\gamma}_{\text{SR}}}}\mathrm{K}_1\left(2\sqrt{\frac{\rho_{\text{APS}}x}{C_{\text{SR}}C_{\text{RD}}\overline{\gamma}_{\text{SR}}\overline{\gamma}_{\text{RD}}}}\right)\right] \quad \text{(A.4)}$$

Substituting Eq. (A.3) and Eq. (A.4) into Eq. (A.2) allows us to write the close-form expression of CDF as Eq. (A.5), where $\mathrm{K}_1(\bullet)$ is first order modified Bessel function of the second kind.

$$F_{\gamma_{\text{end}}^{\text{APS}}}(x)=1-\mathrm{e}^{-\frac{x}{\overline{\gamma}_{\text{RD}}C_{\text{RD}}}}+\frac{1}{\overline{\gamma}_{\text{RD}}C_{\text{RD}}}\mathrm{e}^{-\frac{x}{\overline{\gamma}_{\text{RD}}C_{\text{RD}}}}\left[\overline{\gamma}_{\text{RD}}C_{\text{RD}}-2\sqrt{\frac{\rho_{\text{APS}}\overline{\gamma}_{\text{RD}}C_{\text{RD}}x}{C_{\text{SR}}\overline{\gamma}_{\text{SR}}}}\mathrm{K}_1\left(2\sqrt{\frac{\rho_{\text{APS}}x}{C_{\text{SR}}C_{\text{RD}}\overline{\gamma}_{\text{SR}}\overline{\gamma}_{\text{RD}}}}\right)\right] \quad \text{(A.5)}$$

Appendix B Proof of end-to-end SNR PDF expressions

According to Eq. (17) the derivatives for $F_{\gamma_{\text{end}}^{\text{APS}}}(x)$ of x are as follows.

Divide Eq. (17) into three parts as $F_{\gamma_{\text{end}}^{\text{APS}}}(x)=1-T_1(x)-T_2(x)$. Calculate $T_1(x)$ and $T_2(x)$ derivations respectively as follows:

$$\frac{\mathrm{d}T_1(x)}{\mathrm{d}x}=-\frac{1}{\overline{\gamma}_{\text{RD}}C_{\text{RD}}}\mathrm{e}^{-\frac{x}{\overline{\gamma}_{\text{RD}}C_{\text{RD}}}} \quad \text{(B.1)}$$

$T_2(x)$ derivations can be presented as Eq. (B.2) with the help of [18], where $T_3(x)=-\mathrm{d}T_1(x)/\mathrm{d}x$.

$$\frac{\mathrm{d}T_2(x)}{\mathrm{d}x}=T_3(x)+\frac{1}{C_{\text{RD}}\overline{\gamma}_{RD}}\mathrm{e}^{-\frac{x}{C_{\text{RD}}\overline{\gamma}_{\text{RD}}}}\left(2\sqrt{\frac{\rho_{\text{APS}}x}{C_{\text{SR}}C_{\text{RD}}\overline{\gamma}_{\text{SR}}\overline{\gamma}_{\text{RD}}}}\cdot\mathrm{K}_1\left(2\sqrt{\frac{\rho_{\text{APS}}x}{C_{\text{SR}}C_{\text{RD}}\overline{\gamma}_{\text{SR}}\overline{\gamma}_{\text{RD}}}}\right)+\frac{2\rho_{\text{APS}}}{C_{\text{SR}}C_{\text{RD}}\overline{\gamma}_{RD}}\cdot\right.$$

$$\mathrm{K}_0\left(2\sqrt{\frac{\rho_{\mathrm{APS}}x}{C_{\mathrm{SR}}C_{\mathrm{RD}}\overline{\gamma}_{\mathrm{SR}}\overline{\gamma}_{\mathrm{RD}}}}\right)\Bigg) \tag{B.2}$$

According to the process of derivation above, take $T_1(x)$ and $T_2(x)$ into $F_{\gamma_{\mathrm{end}}^{\mathrm{APS}}}(x)=1-T_1(x)-T_2(x)$, the derivatives of $F_{\gamma_{\mathrm{end}}^{\mathrm{APS}}}(x)$ is finally obtained as Eq. (B.3).

$$f_{\gamma_{\mathrm{end}}^{\mathrm{APS}}}(x)=\frac{2}{C_{\mathrm{SR}}\overline{\gamma}_{\mathrm{SR}}}\mathrm{e}^{-\frac{x}{C_{\mathrm{SR}}\overline{\gamma}_{\mathrm{SR}}}}\left[\sqrt{\frac{\rho_{\mathrm{APS}}x}{C_{\mathrm{SR}}C_{\mathrm{RD}}\overline{\gamma}_{\mathrm{SR}}\overline{\gamma}_{\mathrm{RD}}}}\cdot\right.$$
$$\left[\mathrm{K}_1\left(2\sqrt{\frac{\rho_{\mathrm{APS}}x}{C_{\mathrm{SR}}C_{\mathrm{RD}}\overline{\gamma}_{\mathrm{SR}}\overline{\gamma}_{\mathrm{RD}}}}\right)+\frac{\rho_{\mathrm{APS}}}{C_{\mathrm{SR}}C_{\mathrm{RD}}\overline{\gamma}_{\mathrm{RD}}}\cdot\right.$$
$$\left.\left.\mathrm{K}_0\left(2\sqrt{\frac{\rho_{\mathrm{APS}}x}{C_{\mathrm{SR}}C_{\mathrm{RD}}\overline{\gamma}_{\mathrm{SR}}\overline{\gamma}_{\mathrm{RD}}}}\right)\right]\right] \tag{B.3}$$

References

1. Li F N, Wang D S. Performance analysis of cooperative OFDM network with ordered subcarrier pairing. Proceedings of the 2011 IEEE Wireless Communications and Networking Conference (WCNC'11), Mar 28−31, 2011, Cancun, Mexico. Piscataway, NJ, USA: IEEE, 2011: 1658−1663
2. Suraweera H A, Armstrong, J. Performance of OFDM-based dual-hop amplify-and-forward relaying. IEEE Communications Letters, 2007, 11(9): 726−728
3. Yang L, Qaraqe K, Serpedin E, et al. Performance analysis of amplify-and-forward two-way relaying with co-channel interference and channel estimation error. IEEE Transactions on Communications, 2013, 61(6): 2221−2231
4. Herdin M. A chunk based OFDM amplify-and-forward relaying scheme for 4G mobile radio systems. Proceedings of the 2006 IEEE International Conference on Communications (ICC'06): Vol 10, Jun 11−15, 2006, Istanbul, Turkey. Piscataway, NJ, USA: IEEE, 2006: 4507−4512
5. Kocan, E, Pejanovic-Djurisic M, Javornik T. BER performance enhancement in OFDM AF fixed gain relay systems. Proceedings of the 2013 IEEE EUROCON Conference, Jul 1−4, 2013, Zagreb, Croatia. Piscataway, NJ, USA: IEEE, 2013: 502−507
6. Shah R A, Rajatheva N, Ji Y S. Outage analysis of dual-hop OFDM relay system with subcarrier mapping in Nakagami-m fading. Proceedings of the 2015 IEEE International Conference on Communications (ICC'15), Jun 8-12, 2015, London, UK. Piscataway, NJ, USA: IEEE, 2015: 1807−1812
7. Soliman S S, Beaulieu N C. Exact analysis of dual-hop AF maximum end-to-end SNR relay selection. IEEE Transactions on Communications, 2012, 60(8): 2135−2145
8. Rajkumar S, Thiruvengadam S J. Performance analysis of OFDM based full duplex cognitive AF relay network in the presence of narrowband interference. Proceedings of the 2015 IEEE International Conference on Signal Processing, Informatics, Communication and Energy Systems (SPICES'15), Feb 19−21, 2015, Kozhikode, India. Piscataway, NJ, USA: IEEE, 2015: 5p
9. Rabiei P, Namgoong W, Al-Dhahir N. On the performance of OFDM-based amplify-and-forward relay networks in the presence of phase noise. IEEE Transactions on Communications, 2011, 59(5): 1458−1466
10. Kocan E, Pejanovic-Djurisic M, Karagiannidis G K. On the optimal solution for BER performance improvement in dual-hop OFDM relay systems. Wireless Communications and Networking Conference Workshops (WCNCW'14), Apr 6−9, 2014, Istanbul, Turkey. Piscataway, NJ, USA: IEEE, 2014: 276281
11. Ligata A, Gacanin H, Adachi F. On performance of analog network coding in the presence of phase noise. EURASIP Journal on Wireless Communications and Networking, 2013, DOI: 10.1186/1687-1499-2013-2
12. Fraser D A S. Nonparametric methods in statistics: a Wiley publication in mathematical statistics. 2nd ed. New York, NY, USA: John Wiley & Sons, 1981
13. Simon M K, Alouini M. Digital communication over fading channels. Hoboken, NJ, USA: John Wiley & Sons, 2005
14. Abramovitz M, Stegun I A. Handbook of mathematical functions: with formulas, graphs, and mathematical tables. 9th ed. New York, NY, USA: Dover Publications, 1972
15. Zhang C S, Ge J H, Li J, et al. Performance analysis and power allocation for a two-way amplify-and-forward relay with channel estimation errors. IET Communication, 2012, 6(12): 1846−1855
16. Gradshteyn I S, Ryzhik I M. Table of integrals, series, and products. 6th ed. New York, NY, USA: Academic, 2000
17. Su W, Sadek A K, Liu R. Cooperative communication protocols in wireless networks: performance analysis and optimum power allocation. Wireless Personal Communications, 2008, 44(2): 181−217
18. Dwight H B. Tables of integrals and other mathematical data. 3rd ed. New York, NY, USA: Macmillan, 1957

(Editor: Wang Xuying)