Gram-Schmidt based hybrid beamforming for mmWave MIMO systems

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Abstract

Due to the high cost and power consumption of the radio frequency (RF) chains, it is difficult to implement the full digital beamforming in millimeter-wave (mmWave) multiple-input multiple-output (MIMO) systems. Fortunately, the hybrid beamforming (HBF) is proposed to overcome these limitations by splitting the beamforming process between the analog and digital domains. In recent works, most HBF schemes improve the spectral efficiency based on greedy algorithms. However, the iterative process in greedy algorithms leads to high computational complexity. In this paper, a new method is proposed to achieve a reasonable compromise between complexity and performance. The novel algorithm utilizes the low-complexity Gram-Schmidt method to orthogonalize the candidate vectors. With the orthogonal candidate matrix, the slow greedy algorithm is avoided. Thus, the RF vectors are found simultaneously without any iteration. Additionally, the phase extraction is applied to satisfy the element-wise constant-magnitude constraint on the RF matrix. Simulation results demonstrate that the new HBF algorithm can make substantial improvements in complexity while maintaining good performance.

Keywords MIMO, hybrid beamforming, mmWave

1 Introduction

The capacity of wireless networks should be exponentially increased to meet the explosive demands for ultra high-data-rate transmission. In particular, the upcoming fifth generation (5G) systems aim at carrying out the projected 1000 times or more increase in capacity by 2020 [1]. To achieve this goal, a number of promising technologies have been proposed [2–3]. As a key enabling technology for 5G networks, mmWave communication has gained sudden attractions [4].

MmWave is around and above 30 GHz where the spectrum is less crowded and greater bandwidths are available. Compared with the conventional microwave systems, the main challenges for the success of mmWave systems are the large path-loss and the rain attenuation caused by the unfavorable channel characteristic of mmWave frequency [5]. However, due to the small wavelength of mmWave frequency, large antenna arrays can be packed in a small volume, which allows for an array beamforming technique to combat the large propagation loss with significant beamforming gains. The implement of full baseband design requires a dedicated RF chain consisting of amplifiers, mixers and analog-to-digital or digital-to-analog converters for each antenna [6]. It is impractical for large antenna arrays in mmWave systems. As a result, the HBF combining a digital precoder in baseband and an analog beamformer in RF band is proposed for mmWave MIMO systems to reduce the required number of RF chains [5,7].

Recently, several HBF schemes have been proposed. In Ref. [8] and refined in Ref. [9], the spatially sparse HBF via orthogonal matching pursuit (S-OMP) algorithm is the most widely used HBF algorithm which provides reasonable and good performance [8–9]. It formulates the design problem into a sparsity signal reconstruction problem which is solved approximately by OMP algorithm.
Then in order to reduce the performance gap between the S-OMP hybrid transceiver and the optimal digital design, an iterative HBF design algorithm based on the nonlinear least-squares formulation [10] and an alternating minimization algorithm based on manifold optimization [11] are carried out. However, the two algorithms mentioned above cause a considerably high computational complexity owing to the iterative selection process.

In the domestic research, considering the feasibility of HBF schemes, more recent attention has been turned to reduce the computation complexity of the S-OMP algorithm [12–15]. In Ref. [12], a new method of building the HBF matrixes is proposed, which reduces the computation complexity of the original reconstruction algorithm by reusing the matrix inversion result in each iteration. In Ref. [13], the codebook-based design of hybrid beamformers is presented by using the properties of the matrix containing the array response vectors. Although the codebook-based design provides low complexity, there will be certain performance loss and it is not clear how much the gain can be further obtained. In Ref. [14], an iterative hybrid precoding algorithm was proposed by utilizing the idea of successive interference cancellation (SIC). However, the algorithm is established based on the assumption that the digital matrix is diagonal, which means that the digital beamformer can only allocate power to different data streams. Moreover, the mentioned HBF algorithms all involve iteration process to find the RF matrix or baseband matrix. In Ref. [15], another low-complexity HBF algorithm without iterations is proposed based on discrete Fourier transformation (DFT) codebook, but it has a wide performance gap compared with the S-OMP algorithm.

In this paper, we propose a low complexity HBF algorithm by imposing the orthogonal property of the candidate vectors. Firstly, we prove that the slow greedy process can be avoided when the candidate vectors are orthogonal. To achieve this goal, the low-complexity Gram-Schmidt method is adopted to orthogonalize the candidate vectors, i.e., the array response vectors of the channel. With the orthogonal candidate matrix, the RF matrix can be found parallelly without any iteration. What’s more, the phase extraction is applied to satisfy the element-wise constant-magnitude constraints on the RF matrix imposed by the phase shifters. Compared with the most widely used S-OMP method, the proposed algorithm has a great reduction in the computational complexity. The numerical results demonstrate that the proposed algorithm can achieve a good compromise between complexity and performance.

Notations: throughout this paper, matrices and vectors are set in boldface, with uppercase letters for matrices and lowercase letters for vectors. The superscripts \(^T\), \(^H\), and \(^{-1}\) denote the transpose, conjugate transpose and inverse, respectively. \(\|\) and \(\|\|\) denote the determinant and the Frobenius norm of a matrix, and \(A(:,j)\) and \(A(i,:)\) is used to denote the \(j\)th column and \(i\)th row of the matrix \(A\). The commas and semicolons in a matrix are used as the column and row separator, respectively. Finally, \(I_N\) represents the \(N \times N\) identity matrix.

2 System model and problem formulation

2.1 System model

We consider the single-user mmWave MIMO system shown in Fig. 1. The base station equipped with \(N_r\) antennas and \(L_r\) RF chains sends \(N_t\) data streams to the mobile station with \(N_t\) antennas and \(L_t\) RF chains, where \(N_t \leq L_t \leq N_r\) and \(N_r \leq L_r \leq N_t\). In the hybrid structure, the transmitter applies the \(L_t \times N_t\) baseband precoder \(F_{\text{bb}}\) and \(N_t \times L_t\) RF beamformer \(F_{\text{rf}}\). Notably, both amplitude and phase modification are feasible for the baseband precoder \(F_{\text{bb}}\), but only phase changes can be made to the RF beamformer \(F_{\text{rf}}\) since \(F_{\text{rf}}\) is implemented using analog phase shifters.

![Fig. 1 System model of the HBF design](image)

The received signal at the mobile station can be expressed as

\[
r = \sqrt{\rho}HF_{\text{bb}}F_{\text{rf}}s + n \tag{1}
\]

where \(r\) is the \(N_t \times 1\) received vectors and the scalar \(\rho\) is the transmit power. \(s\) is the \(N_t \times 1\) transmit symbol vector such that \(E[ss^H] = (1/N_t)I_{N_t}\). The total power constraint of the transmitter is satisfied by normalizing \(F_{\text{bb}}\) such that \(\|F_{\text{bb}}F_{\text{bb}}^H\|_2 = N_t\). \(H\) is the channel matrix, and \(n\) is the \(N_t \times 1\) noise vector with independent \(CN(0, \sigma_n^2)\).
1. Entry.

MMWave channels are expected to limited spatial selectivity or scattering because of the high free space path loss of the mmWave bands. For this reason, we adopt a widely accepted narrow clustered channel model based on the extended Saleh Valenzuela model [8–12]. For simplicity of the exposition, each scattering cluster around the transmitter and the receiver is assumed to contribute a single propagation path. With the cluster model, the mmWave channel matrix with L cluster scatters is

\[ H = \sqrt{\frac{N_s N_c}{L}} \sum_{l=1}^{L} \alpha_l a_l(\phi_l, \theta_l) a_l^H(\phi_l, \theta_l) \]  

(2)

where \( \alpha_l \), \( 1 \leq l \leq L \), is the complex gain of the L ray, \( \phi_l(\theta_l) \) and \( \phi_l(\theta_l) \) are its azimuth (elevation) angles of arrival and departure respectively. The vectors \( a_l(\phi_l, \theta_l) \) and \( a_l(\phi_l, \theta_l) \) represent the normalized receive and transmit array response vectors at an azimuth (elevation) angle of \( \phi_l(\theta_l) \) and \( \phi_l(\theta_l) \) respectively.

We assume uniform linear arrays (ULA) in this paper, whose responses do not depend on the elevation angles [9]. For an N-element ULA, the array response vector is

\[ a_{\text{ULA}}(\phi) = \frac{1}{\sqrt{N}} \left[ 1, e^{j2\pi d \sin \phi}, e^{j4\pi d \sin \phi}, \ldots, e^{j(N-1)\pi d \sin \phi} \right]^T \]  

(3)

where \( k = 2\pi / \lambda \) and \( d \) is the inter-element spacing.

The receiver applies the \( N_r \times L_r \) analog combing matrix \( W_{\text{RF}} \) and the \( L_r \times N_r \) baseband combing matrix \( W_{\text{BB}} \). The receive signal \( \tilde{r} \) pre-processed by hybrid combiner is given by

\[ \tilde{r} = \sqrt{P} W_{\text{BB}}^H W_{\text{RF}}^H H_{\text{RF}} F_{\text{BB}} s + W_{\text{BB}}^H W_{\text{RF}} n \]  

(4)

2.2 Problem formulation

Considering that the base station has the perfect channel state information (CSI), we seek to design HBF to maximize the spectral efficiency [9–13]

\[ R = \ln \left( I_{N_s} + \frac{P}{N} R_s |W_{\text{BB}}^H W_{\text{RF}}^H H_{\text{RF}} F_{\text{BB}} F_{\text{BB}}^H F_{\text{RF}} H_{\text{RF}} W_{\text{RF}} W_{\text{BB}} \right) \]  

(5)

where \( R_s = \sigma^2 |W_{\text{BB}}^H W_{\text{RF}}^H W_{\text{RF}} W_{\text{BB}} \) is the noise covariance matrix after combing.

Since maximizing the spectral efficiency involves a joint optimization of four matrix variables with element-wise constant-magnitude constraints on \( F_{\text{RF}} \) and \( W_{\text{RF}} \), it is rather difficult to find the global optimal solutions. Therefore, the design of the joint transmitter-receiver optimization problem is simplified by designing the transmitter and the receiver separately, solving similar optimization problems. In this paper, we focus on the design of HBF at the transmitter and the design of HBF at the receiver is similar and omitted for brevity.

Following the discussion in Ref. [9], near optimal hybrid beamformers can be found by minimizing the Frobenius norm with respect to the optimal unconstrained solution, thus the problem can be formulated as

\[ \left( F_{\text{RF}}^\text{opt}, F_{\text{BB}}^\text{opt} \right) = \arg \min_{F_{\text{RF}}, F_{\text{BB}}} \left\| F_{\text{RF}} - F_{\text{RF}}^\text{opt} F_{\text{BB}} \right\|_F \]  

s.t.

\[ \forall \phi \in \{\phi_i(|\lambda|); 1 \leq i \leq L\} \]  

\[ \left\| F_{\text{RF}} F_{\text{BB}} \right\|_F = N_s \]  

where \( F_{\text{opt}} = [v_1, v_2, \ldots, v_{N_s}] \) is the optimal precoder composed of the right singular vectors of \( H \), which is associated with the largest \( N_s \) eigenvalues obtained by performing SVD on \( H \). \( A \) is a \( N_s \times L_s \) candidate matrix consisting of the array response vectors at the transmitter.

The optimization of the problem in Eq. (6) can be solved with a variant of S-OMP algorithm. However, the S-OMP algorithm [8–9] performs a number of iterations equaling to available RF chains and perform pseudo-inverse in each iteration to find the \( F_{\text{RF}} \) and \( F_{\text{BB}} \), which makes the S-OMP unsuitable for hardware implementation.

In this paper, we first analyze how to utilize the orthogonality of the vectors or atoms in the candidate matrix to avoid the use of the greedy steps and then find a HBF algorithm based on the Gram-Schmidt method.

Apparent complexity reduction is achieved with good performance, as shown in Sects. 3 and 4.

3 Gram-Schmidt based low-complexity HBF algorithm

The main drawback of S-OMP is the matrix inversion and iteration operations which are used to eliminate correlated components from other columns of highly corrected \( A \) [8–9]. In Refs. [13,15], it is simply pointed that the candidate matrix can be set orthogonal to avoid calculating inverse matrix. In this section, we will further analyze how to avoid the slow greedy process by imposing the orthogonal property of the candidate vectors. Moreover, with the orthogonal candidate set obtained by the Gram-Schmidt method, a low-complexity HBF algorithm is proposed.
3.1 Derivation of the proposed HBF algorithm

In S-OMP algorithm, to find the \( L_t \) array response vectors and reconstruct the baseband precoder, the correction matrix between the optimal precoder and candidate matrix is firstly calculated. Then candidate beamforming vector or atom which has maximum correction power with the optimal digital precoder is selected as one of the \( L_t \) columns of \( F_{RF} \). Due to the correction of the columns of the candidate matrix, the residue matrix needs to be updated and the correction matrix should be recalculated.

However, when the candidate matrix is orthogonal, i.e., \( A^H_i A_i = I_L \), we have

\[
(A(:,m))^H A(:,n) = \begin{cases}
0; & m \neq n, \quad 1 \leq m,n \leq L \\
1; & m = n, \quad 1 \leq m,n \leq L
\end{cases}
\]  

(7)

By \( \mathcal{Q} \in \mathcal{Q} = \{1,2,\ldots,L\} \) we denote an ordered subset of cardinality \( k' \) of indices into the candidate vectors. The matrix \((F_{RF})_{k'}\) is composed of the ordered vectors indexed by \( \mathcal{Q}_{k'} \), selected from the candidate matrix \( A_i \) after the \( k' \)th iteration, denoted as \((A_i)^{(k')}\). Thus, the RF matrix \( F_{RF} \) also satisfies the orthogonality, i.e.,

\[
((F_{RF})_{k'})^H (F_{RF})_{k'} = I_{k'}
\]  

(8)

Therefore, in the \( k' \)th iteration, the baseband precoding matrix \( F_{BB} \) which is obtained by calculating the least square solution can be rewritten as

\[
(F_{BB})_{k'} = \left( \left( (F_{RF})_{k'} \right)^{H} \right)^{-1} \left( (F_{RF})_{k'} \right)^{H} F_{opt} = \left( ((F_{RF})_{k'})^{H} \right)^{-1} F_{opt}
\]  

(9)

where the matrix inversion is successfully avoided. The intermediate residual matrix \( F_{res} \) used in the next iteration can be expressed as

\[
(F_{res})_{k'+1} = F_{opt} - ((F_{RF})_{k'}) F_{BB}
\]  

(10)

In its \((k+1)\)th iteration, S-OMP augments the vector index set \( \mathcal{Q}_{k'} := \mathcal{Q}_{k} \cup \{n_{k+1}\} \) by the selection criterion

\[
n_{k+1} := \arg \max_{n_{k+1} \in \mathcal{Q}} \text{diag} \left( \left( \Phi \right)_{k'+1} \left( \left( \Phi \right)_{k'+1} \right)^{H} \right)
\]  

(11)

where \( \left( \Phi \right)_{k'+1} \) is the correction matrix for the \((k'+1)\)th iteration and is given by

\[
\left( \Phi \right)_{k'+1} = A_{i}^{H} \left( F_{res} \right)_{k'+1}
\]  

(12)

Substituting Eqs. (9) and (10) into Eq. (12), the correction matrix \( \left( \Phi \right)_{k'+1} \) can be rewritten as

\[
\left( \Phi \right)_{k'+1} = A_{i}^{H} \left( F_{bb} \right)_{k'+1} = \left( A_{i}^{H} - A_{i}^{H} (F_{RF})_{k'} \left( (F_{RF})_{k'} \right)^{H} \right) F_{opt}
\]  

(13)

Based on the orthogonality of the candidate matrix, we have

\[
A_{i}^{H} (F_{RF})_{k'} \left( (F_{RF})_{k'} \right)^{H} = E_{\mathcal{Q}_{k}} \left( (F_{RF})_{k'} \right)^{H}
\]  

(14)

where \( E_{\mathcal{Q}_{k}} \) is an \( L \times k \) matrix which satisfies

\[
E_{\mathcal{Q}_{k}} = \{E_{\mathcal{Q}_{k}}(l,m), l \in \mathcal{Q}_{k}, m \in (1,k) \} \forall l \in \mathcal{Q}_{k}, m \in (1,k)
\]

(15)

Due to the specificity of the matrix \( E_{\mathcal{Q}_{k}} \), then Eq. (14) can be rewritten as

\[
E_{\mathcal{Q}_{k}} \left( (F_{RF})_{k'} \right)^{H} = E_{\mathcal{Q}_{k}} \left( \left( (A_{i})^{(k')} \right)^{H} \right) = \left( (A_{i})^{(k')} \right)^{H}
\]  

(16)

where \( \left( A_{i} \right)^{(k')} \) is given by

\[
\left( A_{i} \right)^{(k')} = \{ \left( A_{i} \right)^{(k')}(l,l), l \in \mathcal{Q}_{k}, \left( A_{i} \right)^{(k')}(l,l) = A_{i}(l,l); \left( A_{i} \right)^{(k')}(l,l) = 0 \}
\]

(17)

Thus the correction matrix \( \left( \Phi \right)_{k'+1} \) in Eq. (13) is expressed as

\[
\left( \Phi \right)_{k'+1} = A_{i}^{H} \left( F_{BB} \right)_{k'+1} = \left( A_{i}^{H} - \left( A_{i} \right)^{(k')} \right) F_{opt} = \left( A_{i}^{H} - \left( A_{i} \right)^{(k')} \right) F_{opt}
\]

(18)

Eq. (18) shows that the iteration process is simply eliminated the selected columns by setting them zeros and makes no influence on each other, indicating that the \( L_t \) RF columns can be found parallelly and no iterative is needed anymore.

3.2 Gram-Schmidt HBF (GS-HBF)

In Sect. 3.1, we have deduced how to avoid the matrix inversion and iterations based on the orthogonality of the candidate atoms. Thus we consider designing the orthogonal atoms in this section. With these orthogonal atoms, the GS-HBF is presented.

In order to obtain the orthogonal atoms, the low complexity Gram-Schmidt method [16] is utilized to orthogonalize the beamsteering vectors. Based on the orthogonal candidate matrix \( A_i \), the correction matrix \( \Phi \) is given by

\[
\Phi = A_{i}^{H} F_{opt}
\]

(19)

Then the selection criterion of the RF matrix is as follows
\[ F_{\text{RF}} = \left( A \right)^{[a_i]} \]  \hfill (20)

where \( \{ \Omega_i \} \) is the largest \( L_i \) index of \( \text{diag} (\Phi^H\Phi) \).

Therefore, no iterative is needed anymore. Once the RF matrix is obtained, the baseband precoder can be calculated by the least square

\[ F_{\text{BB}} = F_{\text{RF}}^H F_{\text{opt}} \]  \hfill (21)

The baseband precoder is normalized to satisfy the power constraint

\[ F_{\text{BB}} = \frac{F_{\text{BB}}}{\| F_{\text{opt}} F_{\text{BB}} \|} \]  \hfill (22)

As the RF beamforming matrix has the constant-magnitude elements, the final RF matrix can be obtained by extracting the phases of elements in \( F_{\text{RF}} \), i.e.,

\[ F_{\text{final RF}}(i, j) = \frac{F_{\text{RF}}(i, j)}{\| F_{\text{RF}}(i, j) \|} \]  \hfill (23)

Thus the proposed algorithm for the HBF system based on the Gram-Schmidt method contains an orthogonalization step, an HBF step and a normalization step shown in Algorithm 1.

**Algorithm 1 GS-HBF**

Input: \( F_{\text{opt}} \), \( A \), \( N_s, N_c, L_s, L \)  
Output: the HBF matrix \( F_{\text{RF}} \) and \( F_{\text{BB}} \)

The orthogonalization stage

for \( n = 1 \) to \( L \) do

\[ a_n = A(:, n) \]  

for \( m = 1 \) to \( n-1 \) do

\[ a_m = a_m - a_n A(:, n) a_n \]  

end for

\[ a_n = \frac{a_n}{\sqrt{a_n^H a_n}} \]

end for

The HBF stage

\[ \Phi = A^H F_{\text{opt}} \]

\[ \Omega \left\{ \Omega_i : \text{ the largest } L_i \text{ index of diag}(\Phi^H\Phi) \right\} \]

\[ F_{\text{RF}} = \left( A \right)^{[\Omega_i]} \]

\[ F_{\text{BB}} = F_{\text{RF}}^H F_{\text{opt}} \]

The normalization stage

\[ F_{\text{BB}} = \frac{F_{\text{BB}}}{\| F_{\text{opt}} F_{\text{BB}} \|} \]

\[ F_{\text{final RF}}(i, j) = \frac{F_{\text{RF}}(i, j)}{\| F_{\text{RF}}(i, j) \|} \quad i \in [1, N_s], \quad j \in [1, L_s] \]

Return \( F_{\text{final RF}}, F_{\text{BB}} \).

3.3 Complexity analysis

The computational complexity of GS-HBF consists of the following three parts:

1) The first one comes from the Gram-Schmidt orthogonalization, which has a computational complexity of \( O(N_s L) \).

2) The second one is caused by the HBF stage, which requires a computational complexity of \( O(N_s L_i N_s + N_s L_i) \).

3) The third one is from the calculation of normalized matrixes, whose computational complexity is \( O(N_s L_i) \).

Therefore, the proposed HBF algorithm totally requires a computational complexity of \( O(N_s N_c^2 + N_s L_i N_s L_i) \).

The most widely used S-OMP algorithm has a complexity of \( O\left( (L_s)^4 + N_s (L_s)^3 + N_s (L_s) + N_s L_i \right) \), which is mainly caused by the iterative searching process of the RF vectors. Due to the sparse scattering in mmWave environment, the number of the scatters \( L \) generally takes low values and the number of the RF chains should always obey \( N_s \leq L_s \leq L \ll N_s \), which means that the parameters \( N_m, L_s \) and \( N_s \) all have the same orders of magnitude. Hence, compared with the conventional S-OMP algorithm, our proposed algorithm can greatly reduce the computational complexity.

4 Simulation results and analysis

In this section, we present some simulation results of the proposed HBF algorithm GS-HBF simulated under the mmWave MIMO channel model. The narrowband clustered mmWave channel mentioned in Eq. (2) is modeled as \( L=10 \) clusters with uniform distributed azimuth angles of arrival and departure within \([0, 2\pi]\). The complex gains \( a_l \) is generated randomly from a random variable with \( \mathcal{CN}(0,1) \). The interelement spacing of the ULA is assumed to be half-wavelength. The number of the RF chains is equal at the transmitter and receiver, denoted as \( L_s \). For fairness, the same total power constraint is enforced on all HBF schemes and the signal to noise ratio SNR is \( \rho/\sigma^2_s \) range is set to be from \(-40 \) dB to \(0\) dB in all simulations.

In Fig. 2, the GS-HBF is compared against the optimal full digital algorithm, the S-OMP algorithm in Ref. [9] and the low complexity HBF algorithm based on DFT.
codebooks (DFT-HBF) in Ref. [15] when the number of RF chains at both the transmitter and receiver is $L_t = L_r = L_c = 6$ in a $64 \times 16$ massive MIMO system. The situations of transmitting 1, 2 and 4 data streams are examined. It is obvious that the GS-HBF can achieve the approximate performance compared with the S-OMP. The performance of the GS-HBF scheme is close to the optimal algorithm in the cases of $N_s = 1$ and $N_s = 2$ and is within a small gap from the optimality in the case of $N_s = 4$. Moreover, we note that the GS-HBF apparently outperforms the DFT-HBF as the data streams increase.

Moreover, we note that the GS-HBF apparently outperforms the DFT-HBF as the data streams increase.

Fig. 2 Spectral efficiency vs. SNR with different HBF algorithms when $N_s \in \{1, 2, 4\}$

Fig. 3 further demonstrates the spectral efficiency performance by setting the number of transmit data streams to 4 while 6 RF chains are used.

The $64 \times 16$, $128 \times 32$ mmWave MIMO systems are examined. As the array size increases, the performance is evidently improved thanks to the array gain. We see that the proposed GS-HBF achieves spectral efficiencies that are much close to those achieved by S-OMP for both the array settings. Moreover, with array size increasing, the performance gap between the GS-HBF and the optimal algorithm becomes smaller. It is also shown that the proposed algorithm has a better performance as compared to the DFT-HBF algorithm with different array settings.

Fig. 4 shows the spectral efficiency achieved by the GS-HBF algorithm in terms of the number of RF chains. The $64 \times 16$ ULA system is considered with four data streams. In Fig. 4, the performance of the proposed algorithm can sufficiently approach the upper bound given by the rate of optimal fully-digital scheme when the number of RF chains is 7, indicating that the proposed algorithm is robust.

Fig. 4 Spectral efficiency vs. SNR with the proposed GS-HBF algorithm when $L_c \in \{5, 6, 7\}$

5 Conclusions

In this paper, a novel HBF algorithm named GS-HBF is proposed for single-user mmWave systems. The main idea of the GS-HBF is to avoid the greedy process based on the orthogonal candidate vectors. In order to obtain orthogonal candidate vectors, the low-complexity Gram-Schmidt orthogonalization is adopted. Additionally, the phase extraction is applied to satisfy the element-wise constant-magnitude constraints on the RF matrix imposed by the phase shifters. Compared with the S-OMP algorithm, the computational complexity of the GS-HBF is apparently reduced. Simulations show that the proposed HBF algorithm performs rather closely to the existing S-OMP algorithm with reduction in the complexity.
This work was supported by the National Natural Science Foundation of China (61201134), the Hi-Tech Research and Development Program of China (2014AA01A704), and the 111 Project (B08038).

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(Editor: Wang Xuying)