Global optimum encoding packet selection mechanism based on opportunistic network coding for wireless network retransmission

Wang Lian¹², Peng Daiyuan¹, Liang Hongbin³⁴

1. School of Information Science and Technology, Southwest Jiaotong University, Chengdu 610031, China
2. School of Computing Science and Technology, Chongqing University of Posts and Telecommunications, Chongqing 400065, China
3. School of Transportation and Logistics, Southwest Jiaotong University, Chengdu 610031, China
4. School of Computing Informatics and Decision Systems Engineering, Arizona State University, Tempe AZ 85281-8809, USA

Abstract

Packet loss cannot be avoided in wireless network due to wireless transmission medium particularity, therefore improving retransmission efficiency is meaningful to wireless transmission. The current retransmission packet selection mechanisms based on opportunistic network coding (ONC) face low retransmission efficiency and high computational complexity problems. To these problems, an optimized encoding packet selection mechanism based on ONC in wireless network retransmission (OONCR) is proposed. This mechanism is based on mutual exclusion packets and decoding gain concepts, and makes full use of ONC advantages. The main contributions of this scheme are to control the algorithm complexity of the maximum encoding packets selection effectively, avoid the redundancy encoding packets due to the overlapping among encoding packets, and take the encoding packet local and global optimization problem into consideration. Retransmission efficiency is evaluated according to the computational complexity, the throughput, the retransmission redundancy ratio, and the number of average retransmission. Under the various conditions, the number of average retransmission of OONCR is mainly lower than that of other typical retransmission packet selection schemes. The average retransmission redundancy ratios of OONCR are lower about 5%~40% compared with other typical schemes. Simultaneously the computational complexity of OONCR is comparatively lower than that of other typical schemes.

Keywords wireless network, retransmission, network coding, throughput

1 Introduction

Wireless communication is easy to be affected by multipath effect, signal interference, noise etc. Wireless communication has more serious transmission error and packet loss compared with wired network [1–3]. Therefore the reliable transmission has become the principal factor to restrict and affect transmission performance in wireless environment. The reliable transmission technology of wireless network consists of orthogonal frequency division multiplexing (OFDM) [4], multiple input multiple output (MIMO) [5–7], cooperative diversity [8–10], forward error control (FEC) [11–14], automatic repeat request (ARQ) [15–18], hybrid ARQ (HARQ) [19–21] and network coding (NC) etc. NC allows the nodes in the network to encode the incoming packets, and forwards the encoding packets, different from the traditional storage-forward mode. Receivers can recover the requested packets according to corresponding coding packet received and the original packets available, thus realize the maximum transmission capacity deduced by the max-flow min-cut [22–23].

NC technology is not limited to single user packet information to recover the lost packets, instead has taken all the packet information into consideration to recover the lost packets comprehensively, which is apparently superior to point-to-point transmission performance [24]. Therefore
NC technology can recover multiple requested packets simultaneously, which can reduce the number of the retransmission to improve retransmission efficiency. NC technology permits the node to encode the multiple data, which can reach the theory upper bound of multicast.

Network coding can be roughly classified as random linear NC (RLNC) and deterministic encoding. To RLNC, the encoding coefficients are chosen from finite field randomly by the source node [25–27]. To deterministic encoding the encoding coefficients are decided according to the predesigned algorithm by the source node [28–30]. ONC is an important class of deterministic encoding [31–33].

2 Related work

Some classic packet selection algorithms based on ONC have been proposed for wireless network retransmission, such as random-pick [34], most-least [35], NC wireless broadcasting retransmission (NCWBR) [36], Hamming-distance [37], weight-pick [38], weighted ONC retransmission (WONCR) [39] and parallel NC retransmission scheme (PNCR) [40]. In random pick, base station (BS) randomly selects two requested packet to generate encoding packet to the receivers, although this mechanism is simple, but the performance improvement is poor, it has tiny superiority to traditional retransmission schemes. In most-least, BS selects the packet with the most requests and the packet with the least requests to generate the encoding packet. In Hamming-distance, BS selects the two packets with largest Hamming-distance to generate the encoding packet. Most-least and Hamming-distance can have better performance than random-pick, but the number of packets combined or encoded is predetermined and fixed as two. According to the simulation results, the number of packet combined has apparent impact on the final performance [34–36]. The best performance cannot be reached in these two algorithms in most case. In NCWBR, according to the packet distribution matrix (PDM), in each round select the first of ‘1’ of each column in PDM as the packet encoded to generate new encoding. The performance of NCWBR still has tight relationship with packet distribution. When there is more than one packet unknown in the encoding packet to the receiver the requested packet cannot be decoded, and the encoding packet will be deleted. In this case, the performance is declined apparently [36]. Furthermore, some improvement of NCWBR proposed still cannot solve some special case expect to ‘breaking cross relation’, and how to deal redundancy or overlapping among these encoding packets to generate effective encoding packets is still waiting for resolve [41]. NC based retransmission (CoRET) employs Hamming-distance packet selection method and supports multicast wireless broadcast. The goal of CoRET is to find the average optimal number of required packets to encode to achieve the best performance in different channel quality, and the general rule has been proposed [37]. Weight-pick has proposed dynamic combination number and updating lost packet status, but the performance of weight-pick is apparently dependent on the packet distribution, in the extreme status the performance of weight-pick is not stable. Furthermore, the algorithm proposed in original paper is not so cleared and comprehensive [38]. Weighted packet scheduling algorithm based on ONC is proposed for relay-assisted multicast by Gou Liang. This scheme has taken the link state into consideration, and according to weighted PDM (WPDM) to select the packets encoded. This method tries to find the effective encoding packets with maximum number of original packets, but which is based on the searching result of the encoding packet of two original packets encoded with maximum transmission achievement currently. But the local optimum and global optimum of encoding packet problem are not considered in this scheme [39]. PNCR is proposed by using the parallel mechanism to resolve the number of retransmission problem. This method tries to find the effective encoding packets, during the polling procedure when the packet in set $\Omega$ can encode with packets in several stable sets, only the packet in the minimum sequence number stable set is chosen [40]. The following problems are existed some encoding opportunity may be missed when the number of packets encoded is more than 2, and the global optimum problem is not considered.

In the view of above analysis, OONCR proposed in this paper is an effective encoding mechanism. The selection packet mechanism of OONCR is met with the precondition of decoding. OONCR can generate effective encoding packet that can ensure decoding required packets at the receivers. Furthermore, the number of packets combined is dynamically decided according to the packet distribution to get the higher decoding achievement, and there are not additional expenditures on updating packet distribution information. Furthermore this scheme has taken the
redundant encoding packets and the global optimum into consideration.

This paper is organized as follows. Sect. 1 is introduction. In this section, compared with typical retransmission technologies, the advantage of NC applied to wireless retransmission is explained. Further, the typical application scenarios of the retransmission packet selection schemes based on ONC are analyzed. Sect. 2 summarizes the related work. In this section, the disadvantages of typical retransmission packet selection algorithms based on ONC are summarized, and main idea of OONCR is introduced. Sect. 3 is system model and the relative definitions. Sect. 4 is the problem description the encoding packet local and global optimization problem is proposed specifically. The examples are given to illustrate the limitations existed in the maximum encoding packet searching of traditional algorithms. Sect. 5 introduces OONCR scheme. Sect. 6 is mainly the performance analysis including performance evaluation indicator introduction and computational complexity analysis. Sect. 7 is the conclusion of this paper.

3 System model and relative definition

3.1 System model

Typical wireless networks are simplified and abstracted as the following system model, as shown in Fig. 1.

![System model](image)

This system model consists of BS and multiple receivers $R_i (1 \leq i \leq M)$, where BS is responsible to broadcast packets to the end users through wireless channel continually. Due to the non-ideal channel quality and the interference of other radio signals, not all users can successfully receive a packet sent by the BS. When any receiver suffers from packet loss, it requests BS to retransmit. Subsequently BS collects all the requests, and finally the requested packets are transmitted to the receivers by BS. According to the feedback BS can update dynamically the requested packet information. Suppose there are several slots in each transmission cycle, no matter original packet or encoding packet can be transmitted during each slot. Suppose all encoding packets retransmitted can be received and decoded successfully by all the receivers. And the encoding and decoding operation by BS or receivers is binary bitwise XOR.

3.2 Definition

The notation definition is shown in Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>The total number of original packets</td>
</tr>
<tr>
<td>$P_j$</td>
<td>The $j$th original packet</td>
</tr>
<tr>
<td>$R_i$</td>
<td>The $i$th receiver</td>
</tr>
<tr>
<td>$P_{\text{NUM}}(N)$</td>
<td>The number of original packets in PDM</td>
</tr>
<tr>
<td>$\rho(P_j)$</td>
<td>Column vector $P_j (1 \leq j \leq N)$ in PDM</td>
</tr>
<tr>
<td>$w_{ij}$</td>
<td>Weighted value of $p_i$</td>
</tr>
<tr>
<td>$G'_t$</td>
<td>Decoding gain that $P_{st}$ to $R_i$</td>
</tr>
<tr>
<td>$M$</td>
<td>The total number of receivers</td>
</tr>
<tr>
<td>$n_e$</td>
<td>The number of required original packets</td>
</tr>
<tr>
<td>$N'_{\text{LOST}}$</td>
<td>The total number of lost packets of all receivers</td>
</tr>
<tr>
<td>$P_{\text{ENC}}(R_i)$</td>
<td>Row vector $R_i (1 \leq i \leq M)$ in PDM</td>
</tr>
<tr>
<td>$P_{st}$</td>
<td>The $th$ encoding packet</td>
</tr>
</tbody>
</table>

Definition 1 PDM has $N$ columns and $M$ rows, in which each column vector $\rho(P_j)$ stands for the packet $P_j (1 \leq j \leq N)$, while each row vector $\rho(R_i)$ stands for receiver $R_i (1 \leq i \leq M)$. The corresponding item $P_{st}$ in PDM represents the receiving status of the packet $P_j$ to the corresponding receiver $R_i$. When the packet $P_j$ has been received successfully by the receiver $R_i$, $w_{ij}$ is assigned with $w_{ij} = 0$, or else $w_{ij} = 1$.

Definition 2 Decoding necessary condition is at most one requested packet unknown to the corresponding receiver. The requested packet can be recovered according to the encoding packet and the original packets available. Assume the lost packets vectors are $\rho(P_f), \rho(P_g), \ldots, \rho(P_h) (1 \leq f, g, h \leq N)$, all the lost packets can be recovered if and only if there is only one ‘1’ at each row of the matrix.
\( F = [\rho(P_1)^y, ..., \rho(P_g)^y, ..., \rho(P_h)^y] \). Both the encoding and the decoding operations are both XOR.

**Definition 3** Complete decoding means all the lost packets can be decoded to the specified receiver according to the encoding packet and the original packets available. Assume the lost packets vector is \( \rho(P_f), ..., \rho(P_g), ..., \rho(P_h) \) (1 ≤ \( f \), \( g \), \( h \) ≤ \( N \)), the specified packet can be completely decoded if and only if there is no more than one ‘1’ at each row of the matrix \( F = [\rho(P_1)^y, ..., \rho(P_g)^y, ..., \rho(P_h)^y] \). Both the encoding and the decoding operations are both XOR.

**Definition 4** Mutually exclusive packets are these any two packets that are combined into an encoding packet that cannot recover all the lost packets to the specific receivers. Assume the lost packets vectors are \( \rho(P_f), ..., \rho(P_g), ..., \rho(P_h) \) (1 ≤ \( f \), \( g \), \( h \) ≤ \( N \)), the lost packets cannot be recovered completely if more than one ‘1’ at each row of the matrix \( F = [\rho(P_1)^y, ..., \rho(P_g)^y, ..., \rho(P_h)^y] \) occurs, the corresponding packets \( P_f, ..., P_g, ..., P_h \) encoded are called mutually exclusive packets.

**Definition 5** Encoding packet is the encoding packet meets the decoding necessary condition. Effective encoding packet is the encoding packet with new decoding gain, furthermore the new decoding gain is the maximum one to the encoding packets remained. The encoding packets without new decoding gain will be ignored. Additionally the packet can repeatedly occur in encoding packets without additional expenditure.

**Definition 6** Decoding gain \( G' \) refers to the number of the requested packet that has been recovered according to the encoding packet \( P_{\text{fr}} \) and the original packets available at the receiver \( R' \).

\[
G'_i = \sum_{j=1}^{n_q} w_{ij}; \quad 1 \leq i \leq n_q, 1 \leq j \leq N, 1 \leq i \leq M
\]

\[
G'_i = 0; \quad \text{else}
\]  

(1)

**Definition 7** Total decoding gain \( G' \) refers to the sum of the packets recovered according to the encoding packet \( P_{\text{fr}} \) and the original packets available at each receiver.

\[
G' = \sum_{i=1}^{M} G'_i; \quad 1 \leq i \leq n_q, 1 \leq i \leq M
\]

(2)

**Definition 8** The upper bound of retransmission is the maximum number of retransmission. The retransmission upper bound is no more than \( n_q' \), that is the number of required packets.

### 4 Problem description

In this section, the encoding packet local and global optimization problem is proposed specifically. The following given examples are to illustrate the limitations existed in the maximum encoding packet searching of traditional algorithms. ONC is an important deterministic NC [33]. The encoding coefficient is chosen from GF (2), and the simple XOR operation is used. To search for the maximum encoding packets that meet decoding necessary condition, the current retransmission packet selection schemes based on ONC is usually transforming this problem into finding the maximum clique in graph theory. The core idea is transforming the maximum encoding packet problem to the classical maximum-clique problem solution [42–45]. The original packet is mapped to the vertex of undirected graph. That the complete decoding relationship of two packets is mapped to the edge of undirected graph is the vertex unordered pair. The relationships among the packets encoded into the maximum effective encoding are described as the full connected relationship of vertexes in maximum clique, and the recursive method is usually used to find the maximum effective encoding packet. But the following aspects are usually be neglected in this method. Firstly, the overlapping among encoding packets may be existed that leads to the redundant encoding packet. Secondly, the computational complexity of the traditional method is usually high. Thirdly, the most packet selection schemes are only based on local optimization, the global optimization problem is usually not considered, for example random-pick, NCWBR, weight-pick, WONCR and PNCR, etc. The following instances are described these problems and the computational complexity details are analyzed in the performance analysis.

Consider a graph \( G(V, E) \), in which each vertex \( v_i \) (1 ≤ \( i \) ≤ \( n \)) represents the original packet \( P_i \) (1 ≤ \( i \) ≤ \( n \)), and each vertex \( v_j \) ∈ \( V(G) \). For any two different vertexes \( v_a, v_b \) ∈ \( V(G) \), and undirected edge \( E_{ab} \in E(G) \), where \( E_{ab} \) represents the relationship between \( v_a \) and \( v_b \), if \( E_{ab} \) is existed that means \( P_a \) and \( P_b \) are not mutual packets. On the other hand if \( E_{ab} \) is not existed that means \( P_a \) and \( P_b \) are mutual packets that cannot be encoded into one encoding packet. Each packet in PDM is
corresponding to each vertex in the graph, and the set of all vertexes is $V$. If two packets meet the decoding necessary condition, an edge will exist between these two vertexes, and the set of all edges is $E$. Then the connected graph $O(2^n)$ or several connected subgraph can be achieved.

Consequently the problem of search encoding opportunity can be transformed into the problem of grouping the vertexes of the graph. At the premise of decoding necessary condition, to meet the two aspects, the number of group should be least, and any two vertexes in the group should be connected, this problem is NP hard problem, in the worst case the time complexity of the algorithm is $O(2^n)$. In general condition, only the approximate solution can be achieved. For example greedy algorithm can be used to first search for the maximum clique that is complete subgraph with the maximum number of vertexes. Then continue to search for the clique that is complete subgraph with the less number of vertexes until all vertexes have been grouped. Which is the traditional method, but there are still some problems, first searching for maximum clique in graph in fact is NP hard problem, furthermore the searching results maybe not be the optimal solution to the retransmission encoding packets selection.

The following example is to illustrate the problem exited. The PDM is as shown in Table 2. According to the above basic rules, the PDM will be transformed into the undigraph, as shown in Fig. 2. According to the traditional scheme to find the maximum clique in graph theory, the searching result is as shown in Fig. 3. Each subgraph is a complete subgraph, and each subgraph can be taken as one retransmission encoding packet opportunity, each vertex in the subgraph is not mutually exclusive packet. According to the searching result four encoding packets can be achieved:

\[ P_{EP1} = P_1 \oplus P_2 \oplus P_3 \oplus P_4 \]
\[ P_{EP2} = P_2 \oplus P_3 \oplus P_5 \]
\[ P_{EP3} = P_5 \oplus P_6 \]
\[ P_{EP4} = P_7 \]

The following example is to illustrate the problem exited. The PDM is as shown in Table 2. According to the above basic rules, the PDM will be transformed into the undigraph, as shown in Fig. 2. According to the traditional scheme to find the maximum clique in graph theory, the searching result is as shown in Fig. 3. Each subgraph is a complete subgraph, and each subgraph can be taken as one retransmission encoding packet opportunity, each vertex in the subgraph is not mutually exclusive packet. According to the searching result four encoding packets can be achieved:

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\[ P_{EP3} = P_5 \oplus P_6 \]
\[ P_{EP4} = P_7 \]

According to the PDM 1, the requested packet set is $\{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}$, as shown in Table 2. The retransmission encoding packets are $P_{EP1}, P_{EP2}, P_{EP3}, P_{EP4}$. One problem has been ignored that the packets encoded in the encoding packets may be overlapped, not all the encoding packets are effective encoding packets according to the decoding gain. For this example, only three encoding packets $P_{EP1}, P_{EP2}, P_{EP3}$ are needed to recover all the requested packets, the encoding packet $P_{EP4}$ can be ignored.

According to the PDM 2, the requested packet set is $\{P_1, P_2, P_3, P_4, P_5, P_6\}$ as shown in Table 3. The following packet selection scheme can be used. The traditional method is traditional ONC. Four encoding packets are requested in this scheme. The encoding packet $P_{EP1}$ will be chosen because the most requested packets can be recovered. The encoding packet $P_{EP1}$ is the current optimum encoding packet in this scheme. Actually only $\{P_{EP1}, P_{EP2}, P_{EP3}\}$ the three encoding packets are needed to recover all the requested packets when the global optimization has been taken into consideration. The encoding packet $P_{EP3}$ is the local optimum encoding packet in traditional scheme but not the global optimum encoding packet.

<table>
<thead>
<tr>
<th>$R_i$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
<th>$P_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
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<tr>
<td>$R_2$</td>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
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<tr>
<td>$R_3$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$R_4$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$R_5$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$R_6$</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<thead>
<tr>
<th>$R_i$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>$P_6$</th>
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<tbody>
<tr>
<td>$R_1$</td>
<td>0</td>
<td>0</td>
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<td>$R_2$</td>
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<td>$R_3$</td>
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<tr>
<td>$R_4$</td>
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<td>0</td>
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</tr>
<tr>
<td>$R_5$</td>
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<td>0</td>
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<td>0</td>
<td>1</td>
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<tr>
<td>$R_6$</td>
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<td>0</td>
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<td>1</td>
</tr>
</tbody>
</table>
The encoding packets of the traditional method:

\[
\begin{align*}
EP_1 &= P_1 \oplus P_2 \\
EP_2 &= P_3 \\
EP_3 &= P_4 \\
EP_4 &= P_5 \oplus P_6
\end{align*}
\]

The encoding packets of OONCR:

\[
\begin{align*}
EP'_1 &= P_1 \oplus P_2 \\
EP'_2 &= P_3 \oplus P_4 \\
EP'_3 &= P_5 \oplus P_6
\end{align*}
\]

5 OONCR

OONCR focuses on two aspects to optimize traditional scheme. One is to reduce the algorithm complexity to search maximum encoding packets. Another is to improve the retransmission efficiency by optimizing retransmission encoding scheme and taking the local and the global optimization problem into consideration.

In this algorithm OONCR, to reduce the algorithm complexity the maximum number of packets encoded in an encoding packet is set as 4. The maximum number of packets encoded is no more than 4, which is based on the following consideration. First each packet that meets the decoding necessary condition can be encoded together as an encoding packet, and these encoding packets should be decoded by multiple receivers. However the encoding packets that meet the decoding necessary condition will apparently decrease with the number of packets encoded increased. According to the simulation statistics and the former theory analysis, the encoding packet with packets encoded more than 5 is less than \(10^{-6}\), where \(r\) is the ratio between the number of 1 of upward side of main diagonal and the number of item \((m^2-m)/2\) in packet status matrix (PSM) [46]. When the number of packets encoded in maximum encoding packet is more than 4, the probability is very low. With the increment of \(m\) and the number of iterations, the complexity of algorithm will gradually increase, and the corresponding solving time will increase apparently. Meanwhile when \(m\) is more than 4, the increment number of iterations has not apparently affected to the final searching result, on the contrary the loss outweighs the gain. So In this algorithm, the maximum number of packets encoded in maximum encoding packet is set as 4. When is more than 4 the iteration operation will no longer continue. At the premise of the number packet encoded is no more than 4, the time complexity of this algorithm will apparently less than that of greedy algorithm. Secondly, to improve retransmission efficiency the redundancy encoding packets due to the overlapping among encoding packets are removed. And the effective encoding packets are measured according to the corresponding decoding gain. Whether an encoding packet is the final effective encoding packet is depending on the new decoding gain achieved in the global perspective.

5.1 Procedure and algorithm

The main steps of the OONCR algorithm are summarized as follow, based on the definitions mentioned above:

**Step 1** Generate PDM. Each item of PDM \(p_{ij}\) is 0 or 1; when the packet \(p_j\) has been successfully received by the receiver \(R_i\), \(p_{ij}\) is set as 0, or else \(p_{ij}\) is set as 1.

**Step 2** Simplify PDM. If a packet has been successfully received by all the receivers, that is each item of the corresponding column in the PDM is all 0, then delete this column from the PDM. On the contrary if each item of the corresponding column in the PDM is all 1, then delete this column from the PDM, and retransmit the packet directly.

**Step 3** Establish mutual exclusion packets set \(M_{MSR}\).

**Step 4** Simplify mutual packet set: If \(M_{MSR}\) is subset of \(M_{MSR}\), then remove \(M_{MSR}\).

**Step 5** Generate encoding packets set \(S_{EPP}\). Generate \(S_{EPP}\) according to certain sequence of \(P_j\), each packets in \(S_{EPP}\) is not mutually exclusive, that is then packets encoded into the encoding packet \(S_{EPP}\). Because \(S_{EPP}\) is generated according to a certain sequence, the following encoding packet sets do not need to take the packets with less sequence number into consideration. If \(S_{EPP}\) is
empty and $P_j$ is not appeared in any previous encoding packets, then retransmit $P_j$ alone.

**Step 6** Construct multi-elements encoding packet set $P_{EP}$ ($1 \leq n \leq n_3$) of the corresponding packet $P_j$ in sequence according to encoding packets set $S_{EPP}$. According to a certain sequence, first initialize the set $P_{EP}$ with $P_j$ and $P_1$ according to the encoding packets set $S_{EPP}$, if these two packets are existed, or else $S_{EPP}$ is only one $P_j$. If the remaining packet $P_n$ in $S_{EPP}$ not only in $P_{EP}$ but also in $P_{EPP}$ is existed, then add $P_n$ into $P_{EP}$. Continually if the remaining packet $P_n$ in $S_{EPP}$ not only in $S_{EPP}$ but also simultaneously in $S_{EPP}$ and $S_{EPP}$ is existed, then add $P_n$ into $P_{EP}$. The first round of searching process is end, if the packet is not existed or the number of packet in $P_{EP}$ is no less than four. Then continue the next round of search to construct the next encoding packet set.

Algorithm of generating encoding packets set

/*Construct multi-packets encoding packet set $P_{EP}$ ($1 \leq n \leq n_3$) of the corresponding packet $P_j$ */

/* If $1 \leq j, k, s, j \leq N$ */
for $i = 1, n + +, n_3 \leq n_3$
for $j = 1, j + +, j \leq N$
if $S_{EPP} \neq \emptyset$
if $\left( P_j \in S_{EPP} \right)$ and $\left( P_k \in S_{EPP} \right)$
Put $P_j$ and $P_k$ into $P_{EP}$, namely initialize the encoding set $P_{EP}$ ;
if $\left( P_j \in S_{EPP} \right)$ and $\left( P_k \in S_{EPP} \right)$ and
$\left( P_s \in S_{EPP} \right)$
Put $P_s$ into $P_{EP}$, update the encoding set $P_{EP}$;
if $\left( P_j \in S_{EPP} \right)$ and $\left( P_s \in S_{EPP} \right)$
and $\left( P_t \in S_{EPP} \right)$ and
$\left( P_1 \in S_{EPP} \right)$
{Put $P_t$ into $P_{EP}$, update the encoding set $P_{EP}$ ;}
else
Put $P_j$ into $P_{EP}$ ;

**Step 7** Remove redundant encoding packets, and generate effective encoding packets. If each packets of an encoding packet is a subset of another encoding packet, that is $P_{EP} \subset P_{EP}$, then remove $P_{EP}$. If there still are the required packets waiting for retransmission, then take the required packets $P_j$ as an independent encoding packet.

**Step 8** Calculate the total decoding gain $G'$ of corresponding encoding packet $P_{EP}$.

**Step 9** Take the total decoding gain of each encoding packet and the corresponding retransmission encoding packet set into consideration, then decide the final global optimization encoding packet set.

Algorithm of OONCR

/ / $M_{set}$ is the mutual exclusion set to receiver $R$. Each packet in $M_{set}$ cannot occur simultaneously in the encoding packets */
/ / $S_{EPP}$ is the encoding packet set to $P_j$, which means $P_j$ can encode with all the packets in $S_{EPP}$ to generate an effective encoding packet */
/ / $P_{EP}$ is encoding packet $i$ */
/ / $S_{RP}$ is the set of encoding packets */
/ / $S_{RPSET}$ is the set of effective encoding packets set */
/ / $NUMEPSET$ is the number of encoding packets in EPSET */
/ / $N_{ORIG}$ is the mutual exclusion set to receiver */
/ / $S_{RPSET}$ is the set of requested packets, that is the set of requested packets of all receivers */
/ / $S_{RPSET}$ is the set of original packets encoded into the corresponding encoding packet $P_{EP}$ */
/ / $G'$ is the total decoding gain of corresponding encoding packet $P_{EP}$ */

Step 1 Establish mutual exclusion packets set $M_{set}$.
Step 2 Simplify mutual packet set.
Step 3 Generate encoding packets set $S_{EPP}$.
Step 4 Generate encoding packet $P_{EP}$.
Step 5 Remove redundant encoding packets. Initialize $S_{RPSET}$, that is the set of total requested packets. If $S_{RPSET} = \emptyset$
{For $i = 1, i + +, i \leq n_3$
For $j = 2, j + +, j \leq n_3$
If $S_{RPSET} \subseteq S_{EPP}$
$S_{RPSET} \leftarrow S_{RPSET} - P_{EP}$ /*Update $S_{RPSET}$ */

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Initialize \( N_{\text{NUMEPSET}} \)

Step 6 Calculate the total decoding gain \( G \) of corresponding encoding packet \( P_1 \) and choose the encoding packet with the maximum decoding gain as the effective encoding packet successively with dynamically requested packet updating.

DO {
  \{For \( t = 1, t + 1, \ldots, t \leq N_{\text{NUMEPSET}} \) \}
  Calculate the total decoding gain \( G \) according to the current \( S_{\text{EPSET}} \searrow \)
  \( S_{\text{EPSET}} \leftarrow \{ S_{\text{EPSET}} \cup P_1 \} \);
  \( S_{\text{EPSET}} \leftarrow (S_{\text{EPSET}} - P_1) \) /* Update \( S_{\text{EPSET}} \) */ Remove the packets recovered by \( P_1 \) from the previous \( S_{\text{EPSET}} \)
  \( N_{\text{NUMEPSET}} \leftarrow (N_{\text{NUMEPSET}} - 1) \);
} Until \( (N_{\text{NUMEPSET}} = 0) \)

5.2 Case study

An example to illustrate these steps:

Step 1 Generate PDM. As shown in Table 4.

Table 4 PDM of this case

<table>
<thead>
<tr>
<th>( R )</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>( P_4 )</th>
<th>( P_5 )</th>
<th>( P_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( R_3 )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( R_4 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( R_5 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Step 2 Simplify PDM. As shown in Table 5.

Table 5 PDM simplified

<table>
<thead>
<tr>
<th>( R )</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>( P_4 )</th>
<th>( P_5 )</th>
<th>( P_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( R_3 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( R_4 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Step 3 Construct mutual exclusion sets (MS).

\[ M_{\text{MSR}} \searrow \{ P_1, P_3 \} \]
\[ M_{\text{MSR}} \searrow \{ P_2, P_6 \} \]
\[ M_{\text{MSR}} \searrow \{ P_3, P_6 \} \]
\[ M_{\text{MSR}} \searrow \{ P_4, P_5 \} \]
\[ M_{\text{MSR}} \searrow \{ P_5, P_6 \} \]
\[ M_{\text{MSR}} \searrow \{ P_1, P_3 \} \]

Step 4 Simplify mutual exclusion sets.

\[ M_{\text{MSR}} \searrow \{ P_1, P_3 \} \]
\[ M_{\text{MSR}} \searrow \{ P_2, P_6 \} \]
\[ M_{\text{MSR}} \searrow \{ P_3, P_6 \} \]
\[ M_{\text{MSR}} \searrow \{ P_4, P_5 \} \]

Step 5 Construct encoding packet set \( S_{\text{EPSET}} \).

<table>
<thead>
<tr>
<th>( S_{\text{EPSET}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_2, P_3, P_4 )</td>
</tr>
<tr>
<td>( P_3, P_4, P_5 )</td>
</tr>
<tr>
<td>( P_4, P_5, P_6 )</td>
</tr>
<tr>
<td>( P_5, P_6 )</td>
</tr>
</tbody>
</table>

Step 6 Reorganize encoding packet set, and the encoding packet sets of the corresponding \( P_j \) are as Table 6.

Table 6 Encoding packet sets of the corresponding \( P_j \)

<table>
<thead>
<tr>
<th>( P_j )</th>
<th>Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>( { P_2, P_3, P_4 } )</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>( { P_3, P_4, P_5 } )</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>( { P_4, P_5, P_6 } )</td>
</tr>
<tr>
<td>( P_4 )</td>
<td>( { P_5, P_6 } )</td>
</tr>
<tr>
<td>( P_5 )</td>
<td>( { P_6 } )</td>
</tr>
<tr>
<td>( P_6 )</td>
<td>( { P_2, P_3 } )</td>
</tr>
</tbody>
</table>

Step 7 Generate encoding packets.

\[ P_1 = P_3 \oplus P_4 \oplus P_5 \oplus P_6 \]
\[ P_2 = P_1 \oplus P_6 \]
\[ P_3 = P_1 \oplus P_2 \oplus P_5 \]
\[ P_4 = P_1 \oplus P_2 \oplus P_5 \]
\[ P_5 = P_1 \oplus P_4 \]
\[ P_6 = P_1 \oplus P_5 \]

Step 8 Calculate the total decoding gain of each encoding packet.

\[ G_1 = 6 \]
\[ G_2 = 5 \]
\[ G_3 = 3 \]
\[ G_4 = 5 \]
\[ G_5 = 2 \]
G_0 = 4 
G_1 = 4 
G_2 = 6 
G_3 = 6 

**Step 9** Choose the effective encoding packet based on the total decoding gain. And according to the encoding packet chosen and the corresponding decoding gain of the encoding packets left, simplify the encoding packets and decide which encoding packets are the effective encoding packets.

1) \( P'_1 = P'_1 \oplus P'_2 \oplus P'_3 \oplus P'_4 \); effective encoding packet with \( G_{p_1} = 6 \)
   a) \( P'_1 = P'_1 \oplus P'_2 \oplus P'_3 \); \( P_2 \subset P'_1 \), without a new decoding gain, \( P'_2 \) is ignored.
   b) \( P'_2 = P'_1 \oplus P'_3 \oplus P'_4 \); \( P_3 \subset P'_1 \), without a new decoding gain, \( P'_3 \) is ignored.
   c) \( P'_3 = P'_2 \oplus P'_4 \); \( P_3 \subset P'_2 \), without a new decoding gain, \( P'_3 \) is ignored.

According to the recovery information, calculate the total decoding gain of encoding packet left again

2) \( P_2 = P_3 \oplus P_4 \); effective encoding packet with \( G_{p_2} = 6 \)
   a) \( P_2 = P_3 \oplus P_4 \); \( G'_2 = 3 \) and with overlapping with \( P_2 \), \( P_3 \) is ignored.
   b) \( P_3 = P_2 \oplus P_4 \); \( G'_3 = 3 \) and with overlapping with \( P_3 \), \( P_4 \) is ignored.
   c) \( P_4 = P_3 \oplus P_4 \); \( G'_4 = 3 \) and with overlapping with \( P_4 \), \( P_3 \) is ignored.

3) \( P_2 = P_3 \); effective encoding packet with \( G_2 = 6 \), furthermore the \( P_2 \) is only occurred in \( P_2 \)

The final effective encoding packets are \( \{P_1, P_6, P_3\} \).

### 6 Performance analysis

#### 6.1 Computational complexity analysis

The Computational procedure of OONCR consists of several steps in sequence. The Computational complexity of establishing mutual exclusion packets set is \( O(MN) \). \( N \) is the number of original packets, \( M \) is the number of receivers and \( L_i \) is the packet loss ratio of packet \( P_i \). If the original packet received successfully or failed by all receivers, the corresponding packet will be deleted from the PDM at the first step. The Computational complexity will be less than \( O(MN) \). Simplify the mutual exclusion packets set, the final number of the mutual exclusion packets set is no more than \( N \). The core and the most complex calculation step is to construct encoding packet set. In this scheme, the number of encoding packet set is no more than \( n_q (0 \leq n_q \leq N) \). And \( N_{NUM-EP} (0 \leq N_{NUM-EP} \leq N) \) is the number of original packet encoded into an encoding packet. The Computational complexity of traversing the whole encoding packet sets to construct the effective encoding packet is \( O(n_q + N_{NUM-EP}) \). The next step is to calculate the decoding gain to find the final effective encoding packets. Suppose the number of final effective encoding packets is \( N_{NUM-EP} (0 \leq N_{NUM-EP} \leq n_q) \), and the number of receiver is \( M \), the Computational complexity of this step is \( O(N_{NUM-EP} M) \). In all, the total Computational complexity of OONCR is \( O((n_q + N_{NUM-EP} + N_{NUM-EP} M)) \), and the approximate value is \( O(N^2 M) \). The Computational complexity of WONCR is \( O\left( MN^2 \left(1 - \max_{i \in \{1,2,\ldots,M\}} |L_i| \right) \right) \), which has taken the receiver condition into consideration. OONCR can be taken as the special case of WONCR. The main idea of PNCR is to build conflict graph model of loss packet set, each packet in stable set meets the encoding condition. The computational complexity of PNCR is \( O(N^2) \).

#### 6.2 Performance evaluation parameters

In this section, the performance of OONCR is evaluated according to the simulation results. OONCR is compared with the typical schemes in terms of the retransmission efficiency, such as PNCR, weight-pick and random-pick.

The packet loss ratio is defined as the ratio between the lost packets and the total original packets transmitted at each receiver. The total transmission packet is defined as

<table>
<thead>
<tr>
<th>Schemes</th>
<th>The computational complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>WONCR</td>
<td>( O\left( MN^2 \left(1 - \max_{i \in {1,2,\ldots,M}}</td>
</tr>
<tr>
<td>PNCR</td>
<td>( O(N^2) )</td>
</tr>
<tr>
<td>MWC</td>
<td>( O\left( M' N' \left(1 - \max_{i \in {1,2,\ldots,M'}}</td>
</tr>
<tr>
<td>Weight-pick</td>
<td>( O(N^2) )</td>
</tr>
<tr>
<td>Random-pick</td>
<td>( O(N^2) )</td>
</tr>
<tr>
<td>OONCR</td>
<td>( O(N^2 M) )</td>
</tr>
</tbody>
</table>
The summation of all packets transmitted to all receivers. The performance analysis is based on the following indexes: throughput and the number of average retransmission.

1) Throughput is used to evaluate the efficiency of packet transmission.

\[ \eta = \frac{N}{N + \rho} \]  

where \( N \) is the number of original packets, \( \rho \) is the total number of original packets retransmitted.

2) The number of average retransmission is used to evaluate the retransmission efficiency. The retransmission redundancy ratio is the ratio of the redundant number of retransmission to the number of requested packets retransmission.

\[ R_{\text{redundant\_ret}} = \frac{R_A - R_B}{n_q} \]  

where \( R_{\text{redundant\_ret}} \) is the retransmission redundancy ratio, \( n_q \) is the number of requested packets, \( R_A \) and \( R_B \) are the number of retransmission of scheme \( A \) and scheme \( B \) respectively to recover the requested packets, and scheme \( B \) usually is the scheme with the minimum number of retransmission.

6.3 Performance analysis

The performance evaluations are analyzed at the following aspects. Firstly, with the constant loss packet ratio compare the retransmission efficiency in the condition of the varying number of original packets. Secondly, with the constant loss packet ratio compare the retransmission efficiency in the condition of the varying number of receivers. Thirdly, set the constant number of original packets and the receivers, and compare the retransmission efficiency with the varying packet loss ratio.

In the condition of relatively constant packet loss ratio, the number of receivers is 10. The number of original packets is from 5 to 30, and the number of total packet is from 50 to 150. Contrastively, when the packet loss ratio is set as 20%, with the total number of retransmission packets increased, the number of retransmission is overall increased, but that of OONCR is mainly lower than that of other schemes as shown in Fig. 5. Especially the number of retransmission of random-pick and weight-pick are increased rapidly when the total number retransmission packets is increased. The reasons are the number of packet encoded is set as 2 in random-pick, each time the encoding can recover no more than two original packets, so the number of retransmission is comparatively large, and the probability of an effective encoding packets generated according to random-pick scheme is comparatively low. On the other hand, although the number of packets encoded in weight-pick is not limited to 2, but the overlapping among encoding packets is existed, that leads to redundant encoding retransmission packets. Furthermore the retransmission upper bound is not controlled effectively, so the retransmission efficiency is low in the condition of the higher packet loss ratio. OONCR has a slight retransmission efficiency superiority to WONCR and PNCR, the three schemes all use different methods to find the packets that can be encoded into an encoding packet, and all belong to ONC, but WONCR and PNCR are not considered the global optimum of encoding packet. The details have been explained in the related research. OONCR has taken the above limitations into consideration.

![Fig. 5](image_url)  

Comparison of retransmission efficiency with the various number of transmission packets

In the condition of 10 original packets and the number of total packets is from 40 to 280, the packet loss ratio is set as 10%. And the number of receiver is from 4 to 28, as shown in Fig. 6.

![Fig. 6](image_url)  

Comparison of retransmission efficiency with the various number of receivers
Contrastively, with the total number of receivers increased, the number of retransmission is overall increased and comparatively stable, but that of OONCR is mainly lower than that of other schemes.

In the condition of the relatively constant number of original packets and receivers, and varying the discarded packets rate from 10% to 50%. The average retransmission efficiency of different schemes is shown in Fig. 7. As shown in Fig. 7(a), in the condition of 10 original packets and 30 receivers, according to the simulation statistics data, the average retransmission redundancy ratios of WONCR, PNCR, random-pick and weight-pick are 7.75%, 8.18%, 170% and 39.4% respectively compared with OONCR. As shown in Fig. 7(b), in the condition of 12 original packets and 12 receivers, according to the simulation statistics data, the average retransmission redundancy ratios of WONCR, PNCR, random-pick and weight-pick are 11.3%, 5.1%, 198% and 37.9% respectively compared with OONCR.

Contrastively, the number of original packets and the receivers are set as relatively constant values. With the packet loss ratio is increased, the number of retransmission is overall increased, but that of OONCR is mainly lower than that of other schemes. The average number of retransmission of random-pick is obviously the highest with varying packet loss ratio. Because the performance of random-pick is seriously affected by the packet loss distribution, and the probability of an effective encoding packets of random-pick is comparatively low. Furthermore, the number of packets encoded is limited to 2, which limits the decoding achievement of effective encoding packet. The retransmission efficiency of weight-pick is much better than random-pick because the number of packet encoded is dynamically decided according to the current loss packet distribution. However retransmission efficiency of weight-pick is lower than that of WONCR, PNCR and OONCR, the reasons can be concluded as the following. First the overlapping among encoding packets is not considered in weight-pick, and redundant encoding packets are existed. Secondly the retransmission performance of weight-pick is affected obviously by loss packet distribution, in the extreme status its retransmission performance is not stable, and the number of retransmission even will more than the retransmission upper bound. While under the condition of high packet loss rate, the number of retransmission is overall increased. But the retransmission efficiency of OONCR is still lower than that of others. Because OONCR has not only taken the retransmission upper bound control into consideration, but also the retransmission complete encoding packet can recover multiple requested original packets simultaneously, the redundant retransmission encoding packets can be effectively avoided, furthermore the global optimization is also taken into consideration.

7 Conclusions

OONCR is an optimized encoding packet selection mechanism based on opportunistic network coding in wireless network retransmission. Compared with the traditional encoding packets selection algorithm, the simple set operation is adopted, and the upper bound of the number of packets encoded is limited reasonably, which can effectively reduce the algorithm complexity. Second the packet selection scheme has effectively avoided the redundancy encoding packets due to the overlapping among encoding packets to improve retransmission efficiency. To the further improvement, the global
optimization and the local optimization are proposed. Based on the concepts of mutual exclusion packets and decoding gain etc., the effective encoding packets are measured according to the corresponding decoding gain. According to the reorganized multi-elements encoding packet set, multiple complete decoding packets can be achieved. Whether an encoding packet is the final effective encoding packet is depending on the new decoding gain achieved in the global perspective. Subsequently decide the final global optimization encoding packet set. Retransmission efficiency is evaluated according to the throughput, the retransmission redundancy ratio, and the number of average retransmission. The simulation results show OONCR can achieve better retransmission efficiency with the comparatively lower computational complexity compared with other schemes.

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References


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