Outage probability of opportunistic decode-and-forward relaying over Nakagami-$m$ fading channels

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Abstract

In this article, the outage probability behavior of a relay network over Nakagami-$m$ fading channels is analyzed. Both reactive and proactive opportunistic decode-and-forward (DAF) strategies are considered. The closed-form solutions to the outage probabilities on both opportunistic DAF strategies are derived. Simulation results confirm the presented mathematical analysis.

Keywords cooperative communication, opportunistic DAF, Nakagami-$m$ fading, reactive DAF, proactive DAF, outage probability

1 Introduction

The performance wireless communication system will be improved considerably by adopting cooperation among users and exploring the broadcasting nature of the wireless channels [1]. Currently, there are two kinds of cooperative protocols [2–3]. One is based on the amplify-and-forward (AAF) and the other is based on the DAF. In the AAF, the transmitted signal is received by several relay nodes and is amplified directly before being transmitted to the destination nodes. In the DAF, the transmitted signal is decoded by the relay nodes and then is forwarded to the destination nodes. Various opportunistic relaying strategies have been developed [4–7]. However, a majority of performance analysis is on the outage probability under Rayleigh fading channels.

In Ref. [4], a simple opportunistic relaying strategy is adopted and the outage probability under Rayleigh fading channels is derived with both AAF and DAF schemes. However, no closed-from expressions are achieved. In Refs. [8–11], the performance of cooperative communication is discussed with the more general Nakagami-$m$ fading channel model. A closed-form solution is obtained for the AAF relaying strategy.

In this article, the authors extend the work in Refs. [4,9] and obtained the closed-form solution to the outage probability under Nakagami-$m$ fading channels with two opportunistic DAF relaying strategies.

The remainder of the article is organized as follows. The system model is described in Sect. 2. The outage probability analysis is described in Sect. 3 and the simulation results are shown in Sect. 4. Finally, conclusions are presented in Sect.5.

2 System model

A half-duplex dual-hop wireless relay network with multiple relay nodes is studied. As shown in Fig. 1, it consists of one source node ($S$), $M$ relay nodes ($R_k$, $k = 1, 2, \ldots, M$) and one destination node ($D$). From Ref. [4], the source node transmits information and the relay nodes listen to it in the first hop. During the second hop, a relay node is selected to forward its received signal to the destination node.

Here, two different strategies for the DAF protocol are considered, as shown in Fig. 1. In Fig. 1(a), a proactive opportunistic strategy is considered and a reactive strategy is shown in Fig. 1(b). The total system power is limited by $P$. It is assumed that part of the total power $\alpha P$ is allocated to the first hop, where the source node transmits the signal to the relay nodes. The rest of the power is allocated to the second hop, where a selected relay node decodes the received signal and forwards it to the destination node with power $(1-\alpha) P$.

In the proactive opportunistic DAF strategy, the ‘best’ relay is selected before the source transmission according to the
following algorithm:

\[ k_{\text{best}} = \arg \max_{k \in S_{\text{relay}}} \gamma_{k}^{\text{DAF}} \]

where \( \gamma_{k}^{\text{DAF}} = \min \left\{ \alpha |h_{k|s}|^2, (1-\alpha) |h_{k|D}|^2 \right\} \) and the \( R_{k} \in S_{\text{relay}} \) and \( S_{\text{relay}} \) is the set of relay nodes. In this case, as shown in Fig. 1, the channel gain \( h_{sk} \) between the source to the relay and the channel gain \( h_{kD} \) between the ‘best’ relay \( k \) and the destination are assumed to be known.

In the reactive opportunistic DAF scheme, the ‘best’ relay is selected among the set of those successively decoded relay nodes \( D(M) \) during the first hop transmission. Thus, the ‘best’ relay node is selected by

\[ \text{reactive DAF} \]

\[ \arg \max_{k \in D(M)} \gamma_{k}^{\text{DAF}} \]

(2)

With Nakagami-\( m \) fading channels, the channel gain \( h_{sk} \) and \( h_{kD} \) follow the Nakagami-\( m \) distribution with parameters \( (m, \lambda_{s}) \) and \( (m, \lambda_{D}) \) respectively. The variables \( W_{sk} = |h_{sk}|^2 \) and \( W_{kD} = |h_{kD}|^2 \) follow the gamma distributions as

\[ f_{W_{sk}}(w_{sk}) = \frac{(\sigma_{s})^{m_{sk}}w_{sk}^{m_{sk}-1}}{\Gamma(m_{sk})} \exp(-w_{sk}\sigma_{s}) \]

(3)

and

\[ f_{W_{kD}}(w_{kD}) = \frac{\sigma_{D}^{m_{kD}}w_{kD}^{m_{kD}-1}}{\Gamma(m_{kD})} \exp(-w_{kD}\sigma_{D}) \]

(4)

where \( \sigma_{s} = m_{s}/\lambda_{s} \) and \( \sigma_{D} = m_{D}/\lambda_{D} \). Here, it is assumed that all the channels are independent, and the exact channel state information (CSI) is known at the receivers. Meanwhile, it is assumed that the transmitters at the source node or relay nodes have no CSI [9].

3 Outage probability

3.1 Outage probability for the proactive opportunistic DAF

The mutual information of the proactive opportunistic DAF scheme is given as

\[ I = \frac{1}{2} \log \left( 1 + \max_{k \in S_{\text{relay}}} \gamma_{k}^{\text{DAF}} \frac{P}{N_{0}} \right) \]

(5)

where \( N_{0} \) is the noise power. The outage probability is defined as the probability that \( I \) is less than a target rate \( R \), which is denoted as \( P_{\text{out}} = \Pr(I < R) \). Therefore, the outage probability for the proactive opportunistic DAF relaying is obtained as follows [3]:

\[ P_{\text{out,Opp-DAF}} = \Pr \left[ \frac{1}{2} \log \left( 1 + \max_{k \in S_{\text{relay}}} \gamma_{k}^{\text{DAF}} \frac{P}{N_{0}} \right) < R \right] \]

(6)

Let \( \mu = \left( \frac{2^{2R} - 1}{P_{0}} \right)N_{0} \), one has:

\[ P_{\text{out,Opp-DAF}} = \Pr \left[ \max_{k \in S_{\text{relay}}} \gamma_{k}^{\text{DAF}} < \mu \right] \]

(7)

According to the independent assumption of the channels, one has:

\[ P_{\text{out,Opp-DAF}} = \prod_{k=1}^{M} P_{\text{outage}}^{k} \]

(8)

with

\[ P_{\text{outage}}^{k} = \Pr \{ \min \{ \alpha w_{sk}, (1-\alpha)w_{kD} \} < \mu \} = \Pr \{ \alpha w_{sk} < \mu \} + \Pr \{ (1-\alpha)w_{kD} < \mu \} - \Pr \{ \alpha w_{sk} < \mu \} \Pr \{ (1-\alpha)w_{kD} < \mu \} \]

(9)

From Ref. [9], one has:

\[ \Pr \{ \alpha w_{sk} < \mu \} = \int_{0}^{\mu} f_{W_{sk}}(w)dw = \frac{1}{\Gamma(m_{sk})} \Gamma_{w_{sk}}(m_{sk}, \mu\sigma_{s}/\alpha) \]

(10)

where \( \Gamma_{w_{sk}}(\zeta, \tau) \) is the Pearson’s incomplete gamma function. The authors assume that all coefficients \( \{m_{k}\} \) are natural numbers [11]. As discussed in Ref. [9], the following can be obtained:
\[ \Gamma_{\text{inc}}(\zeta, \tau) = \Gamma(\zeta) \left(1 - e^{-\frac{\zeta}{\tau}} \sum_{j=0}^{\infty} \frac{\mu^j}{j!} \right) \]  

(11)

From Eqs. (10) and (11), one has:

\[ \Pr(\alpha w_{\delta} < \mu) = 1 - e^{-\frac{\alpha m}{\mu}} \sum_{j=0}^{\infty} \frac{\mu^j}{j!} \]  

(12)

Obviously,

\[ \Pr(1 - \alpha) \lambda \delta < \mu) = 1 - e^{-\frac{\alpha m}{\mu}} \sum_{j=0}^{\infty} \frac{\mu^j}{j!} \]  

(13)

By placing Eqs. (12) and (13) into Eq. (9), and according to Eq. (8), one has the following outage probability for the Proactive opportunistic DAF with

\[ P_{\text{reactive}_{\text{DAF}}} = \sum_{k=0}^{\infty} \left[ 1 - e^{-\frac{\alpha m}{\mu}} \sum_{j=0}^{\infty} \frac{\mu^j}{j!} \right] + \left[ 1 - e^{-\frac{\alpha m}{\mu}} \sum_{j=0}^{\infty} \frac{\mu^j}{j!} \right] \]

\[ \sum_{j=0}^{\infty} \left( \frac{\mu^{j+1}}{j!} \right) \]  

\[ 1 - e^{-\frac{\alpha m}{\mu}} \sum_{j=0}^{\infty} \frac{\mu^j}{j!} \]  

\[ 1 - e^{-\frac{\alpha m}{\mu}} \sum_{j=0}^{\infty} \frac{\mu^j}{j!} \]  

(14)

3.2 Outage probability for the reactive opportunistic DAF

The authors denote \( D(M) \) as the decoding set of those relay nodes with the ability to fully decode the source message. \( |D(M)| \) is denoted as the number of the set, which is no larger than \( M \). If \( |D(M)| = 0 \), it denotes that there is no relay node in the set \( D(M) \). If \( |D(M)| = M \), it means that \( M \) relays are in the decoding set \( D(M) \). According to Ref. [4], the probability of \( |D(M)| \) is

\[ \Pr(|D(M)|) = C_{\text{M}}^{\text{M}} \prod_{R_k \in D(M)} \Pr \left\{ \frac{1}{2} \ln \left( 1 + \alpha w_{\delta} \frac{P}{N_0} \right) > R \right\}. \]

\[ \prod_{R_k \in D(M)} \Pr \left\{ \frac{1}{2} \ln \left( 1 + \alpha w_{\delta} \frac{P}{N_0} \right) < R \right\} = \]

\[ C_{\text{M}}^{\text{M}} \prod_{R_k \in D(M)} \Pr \left\{ w_{\delta} > \frac{\mu}{\alpha} \right\} \prod_{R_k \in D(M)} \Pr \left\{ w_{\delta} < \frac{\mu}{\alpha} \right\}. \]

(15)

The conditional outage probability for the reactive opportunistic DAF is thus obtained as follows:

\[ \Pr\{|\text{outage}|D(M)\rangle = \left\{ \frac{1}{2} \ln(1 + (1 - \alpha) \frac{w_{\delta} \frac{P}{N_0}}{R_k}) \right\} = \]

\[ \prod_{R_k \in D(M)} \left\{ 1 - e^{-\frac{\alpha m}{\mu}} \sum_{j=0}^{\infty} \frac{\mu^j}{j!} \right\}. \]

(16)

From Eqs. (15) and (16), the outage probability for the reactive opportunistic DAF can be obtained:

\[ P_{\text{reactive}_{\text{DAF}}} = \sum_{k=0}^{\infty} \left[ 1 - e^{-\frac{\alpha m}{\mu}} \sum_{j=0}^{\infty} \frac{\mu^j}{j!} \right] + \left[ 1 - e^{-\frac{\alpha m}{\mu}} \sum_{j=0}^{\infty} \frac{\mu^j}{j!} \right] \]

\[ \sum_{j=0}^{\infty} \left( \frac{\mu^{j+1}}{j!} \right) \]  

\[ 1 - e^{-\frac{\alpha m}{\mu}} \sum_{j=0}^{\infty} \frac{\mu^j}{j!} \]  

\[ 1 - e^{-\frac{\alpha m}{\mu}} \sum_{j=0}^{\infty} \frac{\mu^j}{j!} \]  

(17)

4 Simulation results and discussions

In this section, simulations are presented to test the above theoretical analysis. It is assumed that the Nakagami-\( m \) fading channels are identical for all links of source-relay and relay-destination. That is, \( \mu = m = m \) and \( \lambda = \lambda = 1 \), \( \forall k \in [1, M] \). The power allocation coefficient is set to be \( \alpha = 0.5 \) according to Refs. [4,12]. The target rate threshold \( R \) is set to be 1.

The simulation results are obtained by the Monte Carlo method. For each run, a different channel matrix is generated randomly according to Ref. [13] and 10 000 000 channel matrices are generated totally. Each element of the channel matrix follows the independent Nakagami-\( m \) distribution.
Based on the system model and the relaying strategies, the mutual information is calculated $I$ and the number of $I < R$ is counted. $I < R$ represents occurrence of the outage behavior. The number of $I < R$ divided by $10,000,000$ is the outage probability.

The outage probability of the proactive opportunistic relaying strategy for the cases of different $m$ is shown in Fig. 2. Fig. 3 illustrates the results of the theoretical analysis demonstrated in Eqs. (14) and (17) and the simulation results for the outage of probability of the reactive opportunistic relaying strategy. Both Figs. 2 and 3 show that the theoretical analysis match the simulation results. As expected, with the adoption of more relay nodes, better outage performance is achieved. When $m = 1$, the Nakagami-$m$ fading channel degrades to the Rayleigh fading channel. The results obtained in this article are the same to that in Ref. [4].

5 Conclusions

The closed-form expressions of the outage probabilities for both proactive opportunistic relaying and reactive opportunistic relaying strategies are derived over the Nakagami-$m$ fading channels. The performance of the outage probability is studied. The simulation results verify the theoretical closed-form expressions for the outage probabilities.

References


