EVM simulation and analysis in digital transmitter

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Abstract

The error vector magnitude (EVM) is extensively applied as a metric for digital transmitter signal quality compliance in modern communication systems. This article is focused on the effects of local oscillator (LO) phase noise and nonlinear distortion of power amplifier on EVM. This article contributes to below aspects. First, the relationships between EVM and two effects, LO phase noise and nonlinear distortion of power amplifier, are derived and expressed. Second, to simplify the expression, the third-order intermodulation distortion (IMD3) is used to calculate the EVM. Then, an expression for the EVM is derived based on the digital transmitter model that considers local oscillator phase noise and nonlinear distortion of power amplifier. Finally, the math formula of bit error rate (BER) versus EVM is given which can be easier and more useful to predict BER, according to analysis of the relationship between EVM and signal to noise rate (SNR), inspired by the works of Rishad, Md. Shahriar and AHM, 2006. Simulations are carried out to display the performance of EVM based on these relationships.

Keywords power amplifiers, EVM, phase noise, nonlinear distortion, IMD3

1 Introduction

SNR, BER and EVM are common performance metrics for modern communication systems. Among them, BER performance against SNR is the popular performance criterion used in today’s communication systems. However, with the development of modern communication systems, and the use of different digital modulation types, EVM has gradually become a quite important performance metric to the engineer because it contains information about both amplitude and phase errors in the signal [1–2]. This additional information can allow a more complete picture of channel distortion and is more closely related to the physics [3].

EVM has already been widely used as performance metric of digital transmitter in engineering applications, and it is very useful for the radio designer. While there have been a number of simulations and measurements of EVM [3–5], a general analytical expression relating EVM to the LO phase noise and nonlinear distortion of power amplifier has not been listed, to the author’s knowledge. For example, Ref. [3] only analyzes the influence on EVM with phase noise. The influence on EVM caused by several factors has been studied separately, but it has not acquired the general expression [4]. Authors in Ref. [5] analyze the combined effects of the several factors on EVM in the traditional wireless analog communication system, but the formulas are processed approximately and it is complicated to obtain the value of EVM. In this article, the work is focused on the effects of LO phase noise and nonlinear distortion of power amplifier on EVM based on one type of wireless digital transmitter model. When considering the influence of nonlinear distortion of power amplifier on EVM, a IMD3 is used to calculate EVM which makes the calculation more conveniently and efficiently. Based on the vector composition of phase noise and third-order intermodulation product, improvement has been done for the deduced general expression for EVM compared with Refs. [4–6].

In this article, a digital transmitter model based on the software design radio in Sect. 2 is first briefly introduced. Following this, in Sects. 3 and 4 the relationship between the EVM and two main factors is analyzed, including local oscillator phase noise and non-linear power amplifier. Finally, the general expression to calculate EVM is given when there are both phase noise and non-linear of power amplifier in Sect. 5, and the relationships among EVM, SNR and BER are
analyzed in Sect. 6.

2 Digital transmitter model and EVM

Because of the development of field programmable gate array (FPGA), analog-to-digital converter (ADC), digital-to-analog converter (DAC) and the theoretical maturity of software radio, the communication system is always realized in digital ways. The digital processing part is located as closely as possible to the antenna to minimize the analog elements in digital transmitter.

The model of digital transmitter is comprised of a quadrature modulator, as shown in the block diagram of Fig. 1. The base-band mapping and the digital up-conversion (DUC) are realized by FPGA or digital signal processor (DSP). The modulated intermediate frequency (IF) signal is converted into analog signal through the high-speed DAC. After filtering by the band-pass filter (BPF), the analog IF signal enters into the analog mixer to shift the frequency to radio frequency (RF). The RF signal is transmitted by the antenna after power amplifier.

Fig. 1 Model of digital transmitter

Because of the total digitization of quadrature up-conversion, the transmitter can avoid problems including in-phase and quadrature (I/Q) phase shift deviation, I/Q amplitude imbalance, frequency error, and I/Q zero offset. Therefore, it only needs to consider the effects of LO phase noise and nonlinear distortion of power amplifier on EVM in the digital transmitter.

EVM is an important performance metric of the modulation precision, and EVM is a measure of errors between the measured symbols and expected symbols. EVM represents the dispersion degree of constellation points, and is defined by the following formula [7]:

\[
E_{\text{rms}} = \sqrt{\frac{1}{M} \sum |S_n - S_{n,0}|^2}
\]

where \(E_{\text{rms}}\) represents the root-mean-square (rms) EVM, \(S_n\) is the actual normalized constellation point of the \(n\)th symbol in the stream of measured symbols, \(S_{n,0}\) is the corresponding ideal normalized constellation point of the \(n\)th symbol, and \(M\) is the number of constellation points for different modulation types. For example, \(M = 4\) for quadrature phase shift keying (QPSK) and \(M = 16\) for 16-quadrature amplitude modulation (16 QAM).

3 EVM and local oscillator phase noise

LO phase noise is known as random frequency fluctuations around its center frequency. Phase noise is measured in the frequency domain. There are several ways of defining LO phase noise. The common definition is expressed as a ratio of noise power to carrier signal power, and the noise power is measured in a 1 Hz bandwidth at a given offset from the carrier frequency, and the unit is dBc/Hz. Another definition is the rms phase noise, which can be calculated by the integration of single-sideband phase noise spectrum density in a certain extent of frequency, and the unit is degree. The latter indicates the total phase stability of the LO within the information bandwidth (BW) [5].

The ideal transmitting radio frequency signal without LO phase noise can be expressed as follows:

\[
RF_{\text{ideal}}(t) = I(t)\cos(\omega_c t) + Q(t)\sin(\omega_c t)
\]

where \(I(t)\) is the in-phase baseband signal, \(Q(t)\) is the quadrature baseband signal, and \(\omega_c\) is carrier frequency. If the LO phase noise exists, the transmitting radio frequency signal can be expressed as follows:

\[
RF_{\text{phase}}(t) = I(t)\cos(\omega_c t + \theta(t)) + Q(t)\sin(\omega_c t + \theta(t))
\]

Eq. (3) indicates the phase offset \(\theta(t)\), which introduces a phase shift in the in-band frequencies. In Ref. [3] an equation relating EVM and LO phase noise is given:

\[
E_{\text{rms}} = \sqrt{\frac{1}{E_s/N_0} + 2 - 2\exp\left(-\frac{\sigma^2}{2}\right)}
\]

where \(E_s/N_0\) represents the SNR, \(E_s\) is the energy per symbol, \(N_0\) is the noise power spectral density, \(\sigma\) represents the rms phase noise. Approximating \(\exp(-\sigma^2/2)\) by 2-order Taylor series expansion, Eq. (4) is simplified to:

\[
E_{\text{rms}} = \sqrt{\frac{1}{E_s/N_0} + \sigma^2}
\]

Usually, SNR is very high at the transmitting terminal, thus one can get the effect by phase noise on EVM:

\[
E_{\text{phase}} = E_{\text{rms}} \approx \sigma
\]

Fig. 2 shows the EVM variation with the LO phase noise by
Eq. (5) at different SNR. Fig. 3 shows the EVM variation with the LO phase noise by Eq. (6). Comparing Fig. 2 with Fig. 3, it shows that the curves present an approximate linear growth in Fig. 2 and the slope is roughly consistent with the curves in Fig. 3 when SNR is high. Assuming that SNR is 40 dB, EVM is 2% in Fig. 2 and 1.9% in Fig. 3 when the phase noise is 1°. When the phase noise is 4.5° and SNR is 40 dB, EVM is 8% in Fig. 2 and 7.9% in Fig. 3. Therefore, at high SNR, Eq. (6) can replace Eq. (5) as the simple estimation formula to calculate the EVM caused by LO phase noise.

4  EVM and nonlinear distortion of power amplifier

The conclusions of Ref. [3] indicate that the relationship between EVM and nonlinear distortion of power amplifier can be analyzed through power or Volterra series. The power amplifier of the transmitter usually operates well below its 1 dB compression point. Thus, among the nonlinear effects, the third-order intermodulation interference is the major contribution to EVM. Because IMD3 is the best parameter to characterize nonlinear distortion of power amplifier, IMD3 is used to calculate the EVM to simplify the expression.

The non-linear characteristic of the power amplifier can be expressed by the following series model [8]:

\[ V_o(x) = a_1 x + a_3 x^3 + a_5 x^5 \]  \hspace{1cm} (7)

where \( V_o \) is the RF output signal, \( x \) is the instantaneous RF input signal, \( a_1 \) is the linear gain, \( a_3 \) and \( a_5 \) are the third and fifth order nonlinear coefficients. \( a_1, a_3, \) and \( a_5 \) are decided by the characteristic of power amplifier.

Assume that the input signal is \( v(t) = \sum_{i=1}^{N} A_i \cos(\omega t + \phi_i) \), which means the input signal is cosine carrier without modulation, and \( N \) is the number of the different carriers in the input signal. By inputting it into Eq. (7) and only considering the IMD3 product, the following can be obtained:

\[ V(t) = a_3 \left( \sum_{i=1}^{N} A_i \cos(\omega t) \right)^3 + a_5 \left( \sum_{i=1}^{N} A_i \cos(\omega t) \right)^5 \]  \hspace{1cm} (8)

Consider the case of two tone intermodulation, which means the input signals are two cosine waves with the same amplitude and different frequency. Because of the non-linear characteristic of power amplifier, the third-order intermodulation product \( 2(\omega_i - \omega_j) \) will be obtained when the two carrier signals pass through the power amplifier. According to the conclusions in Ref. [8], the signal amplitude of the third-order intermodulation product is \( \left( \frac{3}{4} \right) a_3 A^2_i \) at the frequency \( 2\omega_i - \omega_j \). The \( I_3 \) is defined as the ratio of this IMD3 product power \( P_3 \) to the carrier power \( P_1 \), and is usually expressed in dBc. The calculation formula is:

\[ I_3 = 10 \log \left( \frac{P_3}{P_1} \right) \]  \hspace{1cm} (9)

Fig. 4 describes IMD3 variation with the normalized input power \( P_{in} \) of IS-IV satellite traveling-wave tube amplifier (TWTA), where the coefficient that \( a_1 = 1.65, a_3 = -0.887, a_5 = 0.16 \) was used [8].

The EVM caused by third-order intermodulation is just the square root ratio of the third-order intermodulation product power to the desired RF carrier signal power [5], as follows:

\[ E_{\text{nonlinear}} = \sqrt{\frac{P_3}{P_1}} = 10^{I_3/20} \]  \hspace{1cm} (10)
Fig. 5 shows EVM variation with IMD3. When considering the influence of nonlinear distortion of power amplifier on EVM, it makes the calculation more conveniently and efficiently because of using IMD3 to calculate EVM. Otherwise, it needs more parameters to calculate EVM, such as input power, output power, the 3rd interception point and the 3rd intermodulation product power, though they are not all easy to obtain.

5 EVM and combined effects

In Eq. (3), while $\theta(t)$ has perturbation, the interference term $\Delta S_{\text{RFphase}}(t)$ caused by LO phase noise can be expressed as follows:

$$\Delta S_{\text{RFphase}}(t) = S_{\text{RFphase}}(t) - S_{\text{RFideal}}(t) = I(t)\cos(\omega_f + \theta(t)) + Q(t)\sin(\omega_f + \theta(t)) - I(t)\cos(\omega_f) - Q(t)\sin(\omega_f) = I(t)[\cos(\omega_f + \theta(t)) - \cos(\omega_f)] + Q(t)[\sin(\omega_f + \theta(t)) - \sin(\omega_f)]$$

$$\theta(t) = \frac{I(t)\cos(\omega_f + \theta(t)) - \cos(\omega_f)}{\theta(t)} + \frac{Q(t)\sin(\omega_f + \theta(t)) - \sin(\omega_f)}{\theta(t)}$$

In the case of slight non-linearity, according to the relationship between the coefficient of the cubic term and the 3rd interception point ($I_{p3}$) [9–10], the series model is modified as follows:

$$y = x - \frac{2x^3}{3I_{p3}}$$

$$I_{p3} = \frac{2a^3_3}{3 a_3}$$

Input Eq. (2) into Eq. (12):

$$S_{\text{RFnonlinea}}(t) = I(t)\cos(\omega_f) + Q(t)\sin(\omega_f) - \frac{2}{3I_{p3}}[I(t)\cos(\omega_f) + Q(t)\sin(\omega_f)]^3$$

In Eq. (14), the component produced by the third term at $\omega_b$ is just the third order intermodulation products. Using product to sum formula to reduce the power of the third term, one can obtain:

$$-\frac{2}{3I_{p3}}[I(t)\cos(\omega_f) + Q(t)\sin(\omega_f)]^3 = -\frac{I^3(t) + I(t)Q^2(t)}{2I_{p3}}
-\frac{Q^3(t) + I^2(t)Q(t)}{2I_{p3}}\sin(\omega_f) + \frac{3I(t)Q^2(t) - I^3(t)}{6I_{p3}}\cos(3\omega_f)
\cos(3\omega_f) + \frac{Q^3(t) - 3I^2(t)Q(t)}{6I_{p3}}\sin(3\omega_f)$$

Hence, the interference $\Delta S_{\text{RFnonlinea}}(t)$ caused by the third order intermodulation at $\omega_b$ can be expressed as follows:

$$\Delta S_{\text{RFnonlinea}}(t) = S_{\text{RFideal}}(t) - S_{\text{RFphase}}(t) - S_{\text{RFnonlinea}}(t) = I(t)^3 + I(t)Q(t)^2\cos(\omega_f) - \frac{Q(t)^3 + I(t)^2Q(t)}{2I_{p3}}\sin(\omega_f)$$

To get the vector relation between the ideal signal vector and the two interference vectors, the interference vector caused by phase noise and the interference vector caused by non-linearity, the baseband signal vector in Eq. (17) can be mapped into I/Q plane. Then, three vector signals are obtained respectively as follows:

$$S_{\text{IQideal}}(t) = (I(t), Q(t))$$

$$S_{\text{IQphase}}(t) = (I(t), Q(t), -I(t)\theta(t))$$

$$S_{\text{IQnonlinear}}(t) = \left( \frac{I(t)^3 + I(t)Q(t)^2}{2I_{p3}}\cos(\omega_f), \frac{Q(t)^3 + I(t)^2Q(t)}{2I_{p3}}\sin(\omega_f) \right)$$

By comparing Eqs. (18)–(20), it is easy to find that interference vector caused by LO phase noise is orthogonal to
the ideal signal vector, while interference vector caused by the third order intermodulation is parallel to the ideal signal vector. Because the two error vectors given by Eqs. (19) and (20) are orthogonal and independent to each other, the total error vector power equals to the sum of the two error vectors’ power. When considering the combined effects of phase noise and the third-order intermodulation interference, the general expression to calculate synthetic EVM is:

$$E_{\text{rms}} = \sqrt{E_{\text{phase}}^2 + E_{\text{nonlinear}}^2}$$

where $E_{\text{phase}}$ can be calculated by Eq. (6) and $E_{\text{nonlinear}}$ can be calculated by Eq. (10).

Fig. 6 shows EVM variation with the combined effects of phase noise and third-order intermodulation interference. From Fig. 6, it is easy to obtain the EVM value of the digital transmitter when the combined effects on EVM including both LO phase noise and nonlinearity of power amplifier are considered.

6 Relationships among EVM, SNR, and BER

It is evident that EVM is essentially the normalized error magnitude between the measured constellation and the ideal constellation [7]. For Gaussian noise model, EVM can also be defined in terms of noise in-phase component, $n_{0,i}$ and quadrature component, $n_{0,q}$ as:

$$E_{\text{rms}} = \sqrt{\frac{1}{T} \sum_{i=1}^{T} (|n_{0,i}|^2 + |n_{0,q}|^2)^{1/2}}$$

where $P_o$ is the power of the normalized ideal constellation or the transmitted constellation, $T$ is the number of symbols used to calculate the mean square value of EVM. For $T \gg M$ ($M$ is the number of constellation points for different modulation types), the ratio of normalized noise power to the normalized power of ideal constellation can be replaced by their unnormalized quantities, that is, Eq. (22) can be modified as follows:

$$E_{\text{rms}} \approx -\frac{1}{2} \left[ \frac{E_n}{N_0} \right]^{1/2}$$

To establish relationship between BER and EVM, SNR in Eq. (23) can be expressed in terms of EVM as [7,11]:

$$\frac{E_n}{N_0} \approx \frac{1}{E_{\text{rms}}^2}$$

By expressing EVM in a logarithmic form, the relationship between SNR and EVM can be obtained as follows:

$$E_{\text{rms}} \approx 20 \log_{10} \left( \frac{1}{\left( \frac{E_n}{N_0} \right)^{1/2}} \right)$$

Considering multi-ary QAM modulation with coherent detection, while carrier frequency and phase are both perfect recovery, the BER of digital multi-ary modulation in Gaussian white noise channel can be shown as [7]:

$$P_b = 2 \left[ 1 - \frac{1}{\sqrt{M}} \right] \text{erfc} \left( \frac{3}{2(M-1)N_0} \right) \left[ 1 - \frac{1}{2} \left( 1 - \frac{1}{\sqrt{M}} \right) \text{erfc} \left( \frac{3}{2(M-1)N_0} \right) \right]$$

where $E_n$ is the energy per bit, and $N_0$ is the noise power spectral density. Defining $E_n/N_0$ as the signal to noise ratio for the $M$-ary modulation system, taking $E_n/N_0 = (E_n/N_0) \text{lb} M$ and $E_n/N_0 \approx 1/E_{\text{rms}}^2$ into Eq. (26), one can now relate the BER directly with the EVM as follows:

$$P_b = 2 \left[ 1 - \frac{1}{\sqrt{M}} \right] \text{erfc} \left( \frac{3}{2(M-1)E_{\text{rms}}^2} \right) \left[ 1 - \frac{1}{2} \left( 1 - \frac{1}{\sqrt{M}} \right) \text{erfc} \left( \frac{3}{2(M-1)E_{\text{rms}}^2} \right) \right]$$

Fig. 7 shows the BER versus SNR performance of different modulation types. Because of the relationship between BER and EVM in Eq. (27), the BER versus EVM curve, as shown in Fig. 8, shows the inverse ratio relationship that exists between BER and EVM (EVM is given by logarithmic form). Comparing Fig. 8 with Fig. 7, it is easy to find that the curves in the two figures are symmetrical with the axis of ordinate, which means EVM (dB) is the minus value of the corresponding SNR (dB) with the given BER. For example, when QPSK modulation is adopted and BER is $10^{-4}$, SNR is 10 dB from Fig. 7, and EVM is $-10$ dB from Fig. 8.
Extended relationships among the bit error rate, signal to noise ratio and error vector magnitude are shown in Figs. 7 and 8. As shown in Figs. 7 and 8, due to normalization, the EVM (dB) is the minus value of the corresponding SNR (dB) with the given BER, and an inverse relationship between them is maintained. Because EVM can be directly measured using vector signal analyzer (VSA), it can save the extra calculation that may be required to find out the BER. Predicting BER with the value of EVM can save the necessary closed loop equipment in the conventional method, and shorten measure time. In addition, according to the relationship between EVM and SNR, SNR can be easily obtained by measuring EVM.

7 Conclusions

In this article, the effects of transmitter imperfections such as nonlinear distortion of power amplifier and LO phase noise on system EVM are analyzed mathematically and graphically. Equations relating EVM to these imperfections are given for radio designers to predict transmitter EVM easily and effectively. These equations can also be used to determine transmitter specifications to meet desired EVM performance. Extended relationships among the bit error rate, signal to noise rate and error vector magnitude are given, then predicting BER with EVM instead of SNR can save the necessary closed loop equipment in the conventional method and shorten the measure time. However, only the relationship between EVM and transmitter imperfections in Gaussian white noise channel is discussed. The effect of different fading channel and that of using EVM-adaptive M-ary modulation systems instead of BER-adaptive systems are now being considered as an extension of the work.

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References


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