Resource allocation based on genetic algorithm for multi-hop OFDM system with non-regenerative relaying

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Abstract

This article investigates resource allocation in multi-hop orthogonal frequency division multiplexing (OFDM) system with amplifying-and-forwarding relaying to maximize the end-to-end capacity. Most existing methods for multi-hop system focus on power allocation or subcarrier selection separately, but joint resource allocation is rarely considered due to the absence of effective interaction schemes. In this work, a novel joint resource allocation methodology is proposed based on Parthenogenic genetic algorithm (PGA), which produces excellent subcarrier allocation set (referred to as individual in PGA) with higher capacity by evolution operator generation by generation. In addition, an adaptive power allocation is also designed to evaluate the fitness of PGA and further enhance the system capacity. Both theoretical analysis and simulated results show the effectiveness of the proposed joint strategy. It outperforms the traditional method by as much as 40% capacity improvement for 3-hop relaying system when system power is high, and obtains much more capacity enhancement percent under conditions of low system power.

Keywords multi-hop OFDM system, non-regenerative relaying, genetic algorithm, power allocation, subcarrier selection, resource allocation

1 Introduction

Earlier works have shown that the system capacity for wireless networks can be improved significantly with the assistance of relaying [1–2]. Hence, for two-hop relaying systems, resource allocation algorithms for two-hop relaying system have been investigated thoroughly. In Refs. [3–4], the system framework included one relay node. In Ref. [3], it is proved that power allocation had a significant impact on system capacity. In Ref. [4], the OFDM technology is employed to relay signals, and an optimal algorithm for subcarrier selection was expressed based on graph theoretical approach. In Ref. [5], the system model was extended to contain two or more relay nodes that transmit parallel signals to the destination. Also, optimal power allocation with outage probability was researched. Considerable attention has been paid to resource allocation for multi-hop relaying network, such as Ad-hoc network and wireless sensor network (WSN). However, these algorithms for two-hop system are not suitable for multi-hop systems because of different feasible solution space. Thus, how to use the OFDM to further benefit relaying in a multi-hop system is still at issue.

Resource allocation for multi-hop relaying OFDM system is generally decomposed into two separate problems, power allocation and subcarrier selection. Ref. [6] proposed an OFDM-based selective relaying scheme, which studied subcarriers selection to deduce the outage probability, but ignored impacts from power allocation. Ref. [7] presented power allocation strategy to OFDM linear multi-hop network with fixed subcarrier selection, aiming to maximize the end-to-end capacity. Ref. [8] developed an optimal power allocation algorithm in the decode and forward (DF) scheme; however, subcarrier selection was simply paired and fixed. Although these algorithms improve the system performance in their tasks, it is desirable that joint allocation of power and subcarrier is used to enhance system capacity. Such joint resource allocation requires a reasonable policy to be able to generate higher capacity through repeated evolution of subcarrier selection and power allocation, which consists with the characteristic of the genetic algorithm (GA). As a powerful optimization tool, the GA has been successfully applied in
OFDM/OFDMA cell network to handle large subcarrier allocation to users without performance degradation in Refs. [9–10].

This article presents a novel resource allocation approach based on PGA to enhance throughputs of multi-hop OFDM relaying system with a total transmission power constraint. By the common master node selection methods in wireless systems, one master node could be selected to collect channel state information (CSI), to implement resource allocation algorithms and to broadcast resource allocation information to other nodes, which makes this resource allocation approach ready to use in practice. The key step of this proposed approach is to generate excellent subcarrier selection population according to PGA, as well as adjust power allocation to facilitate fast convergence. Thus, an adaptive power allocation method is designed to accelerate convergence. The simulation results show that the proposals achieve significant improvement on system capacity.

The remainder of this article is organized as follows. Sect. 2 describes the system model and formulates the capacity maximization problem. In Sect. 3, the novel resource allocation is described based on PGA. In Sect. 4, a new adaptive power allocation is proposed to further improve the performance of resource allocation. In Sect. 5, the performances of the proposed algorithms are evaluated. Finally, conclusions are given in Sect. 6.

2 System model and problem formulation

As shown in Fig. 1, the system contains a source, a destination, and multiple relay nodes. Here, the source is denoted as node 1, the relay node $m$ ($m=1,2,\ldots,M$) is denoted as node $m+1$, the destination is node $M$, $M\geq 2$. It is supposed that the signals from the source cannot be directly received by the destination because of large path loss generated by long distance, and each node $m$ can only detect the signals from the node $m+1$. An OFDM signal is the sum of $N$ independent signals in the frequency domain, which are modulated onto $N$ subcarriers with equal bandwidth $B$. In the amplify-and-forward (AF) scheme, firstly, the node $R_i$ receives the signal $x_n$ from the source node in the $n$th subcarrier, where the received signal are denoted as $y_{n,i} = \frac{p_{n,i}}{E\left[|x_n|^2\right]}x_n + n_{n,i}$ where $E\left[|x_n|^2\right]$ is the power of $x_n$, and $E\left[|x_n|^2\right]=1$ is assumed. $p_{n,i}$ is the power allocated to $x_n$, $h_{n,i}$ is the channel gain of $x_n$ over the first hop. Then, the node $R_i$ chooses one subcarrier and allocates power to forward $y_{n,i}$. Consequently, the signal $y_{n,m}$ including information $x_n$ can also be modulated to different subcarriers at other relaying nodes to maximize the system capacity.

The received $y_{n,m}$ at node $R_m$ ($m=2,\ldots,M$) is expressed by

$$y_{n,m} = \frac{\sqrt{p_{n,m}}h_{n,m}}{\sqrt{E\left[|y_{n,m-1}|^2\right]}}y_{n,m-1} + N_{n,m},$$

Obviously

$$E\left[|y_{n,m}|^2\right] = p_{n,m}h_{n,m} + \sigma^2$$

here, $p_{n,m}$ is the power allocated by the node $R_{m-1}$ to $y_{n,m-1}$, $h_{n,m}$ is channel gain of the $y_{n,m-1}$ at the $m$th hop, and $N_{n,m}$ is additive white Gaussian noise (AWGN), modeled as the same variance $\sigma^2$.

As a result, the received signal including $x_n$ at the destination can be expressed as follows:

$$y_{n,M} = \frac{\sqrt{p_{n,M}}h_{n,M}}{\sqrt{E\left[|y_{n,M-1}|^2\right]}}y_{n,M-1} + N_{n,M} = \frac{\prod_{m=1}^{M} (\sqrt{p_{n,m}}h_{n,m})}{\prod_{m=1}^{M} E\left[|y_{n,m}|^2\right]}x_n + \sum_{m=1}^{M} \frac{\prod_{m=1}^{m-1} (\sqrt{p_{n,m}}h_{n,m})}{\prod_{m=1}^{M} E\left[|y_{n,m}|^2\right]}N_{n,1} + \sum_{m=2}^{M} \frac{\prod_{m=2}^{M} (\sqrt{p_{n,m}}h_{n,m})}{\prod_{m=1}^{M} E\left[|y_{n,m}|^2\right]}N_{n,2} + \ldots + \frac{\prod_{m=M-1}^{M} (\sqrt{p_{n,m}}h_{n,m})}{\prod_{m=1}^{M} E\left[|y_{n,m}|^2\right]}N_{n,M-1} + N_{n,M}.$$  

By Eq. (4), the signal-to-noise ratio of $x_n$ at the destination is expressed as:
the subcarrier selection matrix of an OFDM signal, $H$ is the channel gain of $K$, the following algorithm aims to obtain a good solution $K$ to improve the system capacity. Here, operator $T$ is the real transpose of array or matrix.

2) Population initialization and encoding
At the beginning of running genetic algorithm, a number of individual $K_i$ are randomly generated by exchanging the elements in the same column to form an initial population. The encoding method of $K_i$ is just the numerical value of $K_i$, for example, the structure of genome of $K_i$ is as follows:

$$K_i = \begin{bmatrix}
1 & \cdots & 1 & 1 \\
2 & \cdots & 2 & 2 \\
N & \cdots & N & N \\
\end{bmatrix}$$

3) Fitness and roulette-wheel selection
During each successive generation of general genetic algorithm, a proportion of the existing population is selected to breed a new generation. Individual solutions are selected through a fitness-based process, where fitter solutions (as measured by a fitness function) are typically more likely to be selected. In this joint resource allocation based on PGA, the fitness $f_i$ of individual $K_i$ is defined as system capacity of $K_i$, $f_i=c(K_i)$, the capacity is computed by power allocation method, which is a key point of PGA and will be discussed in next section. According to roulette-wheel of GA, the $K_i$ with high $f_i$ is selected into the new population, $f'_i=f_i/\sum_{j=1}^{I} f_j$, the other $K_i$ is dropped. Here, $I$ is the number of individual $K_i$, $I$ and the number of dropper $K_i$ could be changed by real wireless communication system.

4) Evolution operator
a) Permutation
This step is to generate next generation solutions through evolution operator. According to PGA, the individual produces new one by permuting its own genome. Thus, $K_i$ permutes its element $K_i(n,m)$, which is at the nth row and the mth column of $K_i$, with the element $K_i(n',m)$ to generate a new subcarrier allocation solution, $m$ and $n$ are both selected randomly, $n'\in [n+1,N]$. If the fitness of $K'_i$ is no less than that of $K_i$, the permutation is effective, or the number $n'$ is added by one. The permutation is repeated until $n'=N$. Once the permutation is effective, this new solution $K'_i$ is put to current population. An example of permutation is as below:

$$\begin{align}
\prod_{n=1}^{M} (p_{n,m}h_{n,m}) & \prod_{n=1}^{M} E[y_{n,m}]^2 + \prod_{n=2}^{M} E[y_{n,m}]^2 + \cdots + \\
\prod_{n=M-1}^{M} (p_{n,m}h_{n,m}) & \prod_{n=M-1}^{M} E[y_{n,m}]^2 \\
\prod_{n=M-1}^{M} (p_{n,m}h_{n,m}) & \prod_{n=M-1}^{M} E[y_{n,m}]^2 \\
\end{align}$$
\[
(k_{n,1} \cdots k_{n,m} \cdots k_{n,M}) \quad \Rightarrow \quad (k'_{n,1} \cdots k'_{n,m} \cdots k'_{n,M})
\]
\[
(k_{x,1} \cdots k_{x,m} \cdots k_{x,M}) \quad \Rightarrow \quad (k'_{x,1} \cdots k'_{x,m} \cdots k'_{x,M})
\]

**b) Mutation**

The idea behind mutation is to prevent the solution falling to a local minimum. After all \( K_i \) finish their permutation operator, several \( K_i \) randomly exchange its two elements in the same column again, these \( K_i \) and related columns are randomly produced.

Finally, the algorithm goes to step 3) to select solutions with high \( f_i \) by roulette-wheel selection method. As a result, a new population with higher capacity is generated.

5) Termination criterion

After the phase of evaluation and selection, the times of iteration \( t \) is added by one. The iterations will continue until the iteration times \( t = T \), where \( T \) is an integer for a predefined threshold. The larger \( T \) is, the more running time is needed for PGA and the better PGA works. In the final population, the \( K_i \) with highest capacity gives the best subcarrier selection, the power allocation for evaluating the fitness of this \( K_i \) gives the power distribution to every \( x_n \) and to every hop of \( x_n \).

From the description above, the PGA process flow is shown in Fig. 2. The computing complexity of this joint resource allocation is determined by the iteration times \( T, I \), which is the number of the individual in the population, and the permutation times. The permutation times is random, but it is not larger than the subcarrier number \( N \). Hence, \( O(TINP) \) is given as the upper bound on computing complexity of this joint resource allocation, and \( O(P) \) denotes the computing complexity of power allocation method used to evaluate the fitness of each individual.

**Fig. 2** The work flow of joint resource allocation based on PGA

4 **Adaptive power allocation**

In this resource allocation algorithm based on PGA, power allocation plays an important role. It determines the fitness of every individual. A good power allocation can evaluate the individual more correctly, and improve system capacity. In this article, the usual averaged power allocation method is dropped, and the power allocation method for an individual is decomposed to two steps. First, the authors derive the optimal power \( P_{n,n} \), when power \( P_n \) is fixed, \( P_n \) is the sum of
\( p_{n,m}, m \in \{1, 2, \ldots, M\} \). Secondly, they find the power allocation of \( p_n \) under system power constraint.

4.1 The optimal power \( p_{n,m} \) for fixed \( p_n \)

If \( p_n > 0 \), then \( p_{n,m} > 0 \), otherwise \( z_n = 0 \), the power \( p_n \) is wasted. Thus, when \( p_n > 0 \), Eq. (5) is transformed to Eq. (7).

\[
\begin{align*}
\gamma_n & = \frac{1}{\sum_{m=1}^{M} \frac{1}{p_{n,m} h_{n,m}^+}} - 1 \\
& = \frac{1}{\sum_{m=1}^{M} \frac{1}{p_{n,m} h_{n,m}^+}} - 1 + \frac{\sum_{m=1}^{M} \frac{1}{p_{n,m} h_{n,m}^+}}{\sum_{m=1}^{M} \frac{1}{p_{n,m} h_{n,m}^+}} - 1 \\
& = \frac{\sum_{m=1}^{M} \frac{1}{p_{n,m} h_{n,m}^+}}{\sum_{m=1}^{M} \frac{1}{p_{n,m} h_{n,m}^+}} - 1.
\end{align*}
\]

By the mathematical induction method (see Appendix A), Eq. (7) is equivalent to Eq. (8).

\[
\begin{align*}
\gamma_n & = \frac{1}{\sum_{m=1}^{M} \frac{1}{p_{n,m} h_{n,m}^+}} - 1 \\
& = \frac{\sum_{m=1}^{M} \frac{1}{p_{n,m} h_{n,m}^+}}{\sum_{m=1}^{M} \frac{1}{p_{n,m} h_{n,m}^+}} - 1 \\
& = \frac{\sum_{m=1}^{M} \frac{1}{p_{n,m} h_{n,m}^+}}{\sum_{m=1}^{M} \frac{1}{p_{n,m} h_{n,m}^+}} - 1.
\end{align*}
\]

Now, the optimal \((p_{n,1}, \ldots, p_{n,m}, \ldots, p_{n,M})\) is the solution that minimizes the denominator of Eq. (8) based on the optimization theory, which can be obtained by Lagrange Multiplier algorithm. The analysis is as follows:

\[
\begin{align*}
& f(p_{n,1}, \ldots, p_{n,m}, \ldots, p_{n,M}) = \prod_{m=1}^{M} \left( \frac{\sigma^2}{p_{n,m} h_{n,m}^+} + 1 \right) - 1 + \\
& \lambda(p_{n,1}, \ldots, p_{n,m}, \ldots, p_{n,M} - p_n)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial f}{\partial p_{n,M}} & = \prod_{m=1}^{M} \frac{\sigma^2}{p_{n,m} h_{n,m}^+} + 1 - 1 + \\
& \lambda = 0 \\
\vdots
\end{align*}
\]

By Eq. (9), one can obtain the following equation:

\[
\begin{align*}
\frac{\partial f}{\partial p_{n,m}} & = \prod_{m=1}^{M} \frac{\sigma^2}{p_{n,m} h_{n,m}^+} + 1 - 1 + \\
& \lambda = 0 \\
\vdots
\end{align*}
\]

Until now, the adaptive power allocation for multi-hop OFDM systems in the AF scheme is completed. An efficient
algorithm to obtain the optimal power \( \{P_{n,1}, \ldots, P_{n,n}, \ldots, P_{n,M} \} \) for every hop of \( x_n \) is developed by Sect. 4.1, and an effective power \( P_n \) for each \( x_n \) is obtained by Sect. 4.2.

5 Simulation results

Monte Carlo simulation results are shown to verify the efficient performance of the proposed joint resource allocation strategy. In this simulation, the propagation loss factor is set to 3 and the shadow effect is not considered, the path loss between node (0,0) and node (100, 0) is supposed to be 50 dB, thus, the path loss between two nodes with distance \( d \) is \( 50 + 3 \times 10 \log(d/d_0) \) (dB), \( d_0 \) is 100. Three-path Rayleigh fading channel is included, each multipath component is modeled by Clarke’s flat fading model, it is assumed that the power of each delayed path is exponentially attenuated, and the mobile speed is 3 km/h. Specific values of the simulation parameters are shown in Table 1. The number of individuals is 30. The iterative times \( T \) is 100 from Fig. 3 to Fig. 5.

Table 1 Simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path loss</td>
<td>( 50 + 3 \times 10 \log(d/d_0) ) dB</td>
</tr>
<tr>
<td>( d_0 )</td>
<td>100</td>
</tr>
<tr>
<td>Central frequency</td>
<td>2 GHz</td>
</tr>
<tr>
<td>Number of subcarriers</td>
<td>64</td>
</tr>
<tr>
<td>Bandwidth of each subcarrier</td>
<td>15 kHz</td>
</tr>
<tr>
<td>Total bandwidth</td>
<td>0.96 MHz</td>
</tr>
<tr>
<td>The mobile speed/Maximum Doppler shift</td>
<td>3(km⋅h⁻¹)/5.56 Hz</td>
</tr>
<tr>
<td>Noise power</td>
<td>-50 dBm</td>
</tr>
<tr>
<td>Power delay profile</td>
<td>([0, e^{-1}, e^{-2}])</td>
</tr>
<tr>
<td>Power attenuation parameters</td>
<td>([0, 2.5, 5]) µs</td>
</tr>
</tbody>
</table>

Fig. 3 Comparison of four algorithms for 3-hop

Fig. 3 gives the normalized system capacity \((\text{bit} \cdot \text{s}^{-1} \cdot \text{Hz}^{-1})\) with system power \( P \) changed from -2 dBm to 20 dBm. It is a 3-hop system with node 1–4 being located at (0,0), (33,0), (66,0), (100,0). According to the proposed joint resource allocation algorithm and adaptive power allocation method, the evaluated algorithms include: Alg. 1, the averaged power allocation with fixed subcarrier allocation (FSA); Alg. 2, the proposed adaptive power allocation with FSA; Alg. 3, the joint resource allocation (JRA) based on averaged power allocation; Alg. 4, JRA based on the proposed adaptive power allocation. Here, the power allocation methods in Alg. 3 and Alg. 4 are used to evaluate the individual fitness, which is an important criterion to put the excellent subcarrier sets into the next generation population in JRA. From Fig. 3, compared with Alg. 1, Alg. 2 improves capacity significantly, which validates the performance of adaptive power allocation. Based on the PGA, the joint resource allocation Alg. 3 is an effective way to enhance system capacity; it could generate better subcarrier selection set than Alg. 1 under the same power allocation method. Hence, along with the proposed adaptive power allocation method proved by Alg. 2, Alg. 4 works best under the whole power condition. The differences between Alg. 4 and Alg. 3 indicate the importance of evaluating fitness for individual, the evaluating fitness with adaptive power allocation method is more reasonable than the averaged power allocation.
Figs. 4 and 5 give illustrate the impacts the hop number has on the parameters averaged $z_n$ and normalized system capacity. These two parameters are computed by the proposed Alg. 4. The power is from $-2$ dBm to 20 dBm, the nodes are equally distributed from (0,0) to (100,0) in x-axis for every multi-hop system. In Under the same system power condition, Fig. 4 has shown that the averaged signal-to-noise ratio $z = \frac{\sum z_n}{N}$ is improved greatly considerably with the ascending hop number. In Fig. 5, the order of hop numbers is not fixed according to capacity in the different power conditions, when the system power is 10 dBm, the 3-hop capacity is the lowest, while it is higher than 4-hop and 5-hop systems when the power is 18 dBm. However, normalized system capacity has shown a downward trend along with increasing hop number under the constraint of high system power, because multi-hop system consumed much more time, which counteracts the positive effect from signal-to-noise ratio $z$. The optimal hop number decreases as the system power increases. This result is similar to the one with DF scheme in Ref. [7].

To make out the convergence of joint resource allocation based on different power methods for evaluating fitness, including the averaged power allocation method and the proposed adaptive power allocation method, Fig. 5 is used to express the convergence rate, when total power of 3-hop OFDM relay system is 20 dBm. From Fig. 6, the algorithm based on adaptive power allocation method achieves its convergence point when evolution times is near to 80, while the capacity enhancement of the algorithm with equal power allocation becomes slow after it arrives at 70, the difference of these two convergence points is less than 10 times. In addition, during the evolution process, the JRA algorithm based on proposed adaptive power allocation can evaluate the fitness of new individual more correctly, the capacity gain caused by adaptive power allocation is approximately 0.12 bit s$^{-1}$ Hz$^{-1}$, which is higher than the capacity gain 0.07 bit s$^{-1}$ Hz$^{-1}$ produced by equal power allocation method. It is clear that an excellent fitness evaluation method for joint resource allocation is very important, and the proposed adaptive power allocation makes significant contributions to system capacity.

6 Conclusions

In this article, a joint resource allocation for multi-hop OFDM relaying system is proposed for increasing the system performance. A combined allocation is implemented as follows. The subcarrier set is evolved according to PAG, and power allocation is carried out to evaluate the fitness of population, which is the criterion of subcarrier selection set. To further improve system capacity, an adaptive power allocation is also derived by Lagrange Multiplier algorithm and Greedy algorithm. The simulation results show that, compared to equal power allocation and traditional subcarrier selection algorithms, the proposed strategy improves system capacity significantly as a result of adaptive power allocation and joint optimization by PAG. It is demonstrated that more hops can improve the signal-to-noise ratio as well, but it may not result in high capacity since multi-hop inevitably enlarges time-consumption.

Future work may involve how to further reduce the computing complexity of the proposed algorithm, and the joint resource allocation in multi-hop OFDM system with regenerative relaying is also under consideration.

Acknowledgements

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Appendix A

The authors’ objective is to prove that Eq. (A.1) equals to Eq. (A.2).

$$\frac{\sum \sigma^2}{p_{s,n}h_{s,n} + \sum \sigma^2} \cdots + \frac{\sum \sigma^2}{p_{s,n}h_{s,n}} + \frac{\sum \sigma^2}{p_{s,n}h_{s,n}} \cdots + \frac{\sum \sigma^2}{p_{s,n}h_{s,n}} = \prod_{n=1}^{M} \left( \frac{\sum \sigma^2}{p_{s,n}h_{s,n}} + 1 \right) - 1$$

(A.1)

$$\prod_{n=1}^{M} \left( \frac{\sum \sigma^2}{p_{s,n}h_{s,n}} + 1 \right) - 1$$

(A.2)
Proof

For $M = 2$, obviously, Eq. (A.1) equals to Eq. (A.2).

Now, when $M = m$, $m \geq 3$, the following Eq. (A.3) is supposed:

$$\sum_{m=1}^{m-1} p_{n,m} h_{n,m} + \sigma^2 \prod_{m=1}^{m-1} p_{n,m} h_{n,m} = \prod_{m=1}^{m-1} \left( \frac{\sigma^2}{p_{n,m} h_{n,m}} + 1 \right) - 1$$

(A.3)

When $M = m + 1$, Eq. (A.1) is expressed as follows:

$$\sum_{m=1}^{m} p_{n,m} h_{n,m} + \sigma^2 \prod_{m=1}^{m} p_{n,m} h_{n,m} = \prod_{m=1}^{m} \left( \frac{\sigma^2}{p_{n,m} h_{n,m}} + 1 \right) - 1$$

(A.4)

By the Eq. (A.3) when $M = m$, Eq. (A.4) could be reformulated to Eq. (A.5)

$$\sum_{m=1}^{m} \left( \frac{\sigma^2}{p_{n,m} h_{n,m}} + 1 \right) - 1 + \sum_{m=1}^{m} \prod_{m=1}^{m} \left( \frac{\sigma^2}{p_{n,m} h_{n,m}} + 1 \right) = \prod_{m=1}^{m} \left( \frac{\sigma^2}{p_{n,m} h_{n,m}} + 1 \right) - 1$$

(A.5)

Thus, according to above mathematical induction method, for each $M$, $M \geq 2$, Eq. (A.1) equals to Eq. (A.2).

Proof is completed.

References


(Editor: WANG Xu-ying)