Performance analysis of amplify-and-forward relaying with hybrid-ARQ in Rayleigh fading channels

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Abstract

Hybrid automatic repeat request (HARQ) is a well-known technique for improving system throughput and link performance of wireless communication systems, including cooperative communication systems. The amplify-and-forward (AF) relaying method is one of the most attractive cooperative diversity schemes because of its low complexity. In this article, the end-to-end performance in terms of block error rate (BLER) and normalized throughput of AF relaying with HARQ transmission under the Rayleigh fading channel is analyzed. Numerical results validate the proposed analysis and demonstrate the gain of HARQ schemes in AF relaying systems. This analytical method can be extended to the systems with other HARQ protocols and other cooperative relaying schemes.

Keywords cooperative diversity, AF, HARQ, BLER

1 Introduction

Cooperative communication provides a new way of introducing spatial diversity by creating a virtual antenna array (VAA) in wireless systems, where the mobile stations may not be able to support multiple antennas due to size or other constraints. Several repetition-based cooperative diversity algorithms such as AF and decode-and-forward (DF) are developed to fully exploit the spectral diversity for reducing the outage probability [1]. Meanwhile, HARQ has been adopted as one key technique for improving the system throughput in wireless communication systems such as 3G long-term evolution (LTE) [2]. Hence, the performance of HARQ protocols in cooperative systems has attracted much research interests. In Ref. [3], Monte Carlo simulations are used to evaluate the long term average throughput in relay networks. The outage probability of a HARQ protocol for relay channels is studied in Ref. [4]. Also, the throughput-delay performance of a half-duplex relay channel with HARQ is analyzed in Ref. [5], whose protocol uses a form of incremental redundancy HARQ transmission with assistance from the relay via space-time coding.

Most of earlier works have been focused on analyzing the capacity or outage for cooperative networks, which only provides the theoretical performance bound. However, no BLER performance of HARQ in cooperative systems has been investigated so far, which is exact measurement metric of the practical systems. Then, in this article, the authors study the error rate performance and corresponding normalized throughput of a dual-hop AF relaying system with HARQ transmission under quasi-static Rayleigh fading channels. The union bounds are derived and validated by the simulation results. This analysis method can be easily extended to the scenarios with other relaying schemes and HARQ protocols.

2 System overview

2.1 Protocol description

Throughout this article the half-duplex relay is assumed for the sake of implementation. This is because for the same
antenna element, the received and outgoing signals may vary in a very large range, which renders it difficult for practical implementation of terminals that transmits and receives in the same band simultaneously. Both time and frequency can be used to separate the receiving and transmitting phases. For simplicity, a protocol is adopted by which the two phases are orthogonal in the time domain. Fig. 1 illustrates the procedures of HARQ transmission in AF relaying channel. During each HARQ transmission round, $K$ information bits are encoded into code-words of length-$N$ and transmitted by the source to the relay and destination in the first time slot. It is assumed that each codeword corresponds to one packet. In the first transmission round, the relay just transmits the data between the source and the destination with power normalization in the second time slot. When the retransmission happened, the relay combines the current signal with those transmitted in the previous transmission rounds using maximal ratio combining (MRC) before forwarding. At the end of each transmission round, the destination decodes the packet and detects the error through the cyclic redundancy check (CRC). Then, the transmission result is broadcasted through a 1-bit acknowledge (ACK) or non-acknowledge (NACK) message. The NACK/ACK is assumed to be received error-free with negligible delay. As long as NACK is received after each HARQ round and the maximum number of HARQ rounds is not reached, the source successively retransmits the packet containing the same information bits. Otherwise, the subsequent packets are transmitted by the source.

This end-to-end HARQ mechanism is simple and deals with handover easily, because the source knows the status of transmitted HARQ blocks. However, it also has drawbacks such as low transmission efficiency and long transmission delay [6].

2.2 System model

As shown in Fig. 2, consider a simple wireless cooperative communication system in which a relay $R$ node cooperates with a source $S$ node to transmit information to a destination $D$ node. Perfect channel state information (CSI) knowledge is assumed to be known at the receiver. As in Ref. [5], all links are assumed to be long term quasi-static wherein all HARQ rounds of a single packet experience a single channel realization. Subsequent packets experience independent channel realizations.

The signal transmitted by the source during the first time slot is denoted as $x_i$. It is assumed that $E[x_i] = 0$ and $E[|x_i|^2] = P_s$, where $P_s$ is the average transmitted power. During the $k$th transmission round, the signals received at $R$ and $D$ at the first time slot are written by

$$
\begin{align*}
    y_{R,k}^{(1)} &= h_{S,R} x_i + z_{R,i}^{(1)} \\
    y_{D,k}^{(1)} &= h_{S,D} x_i + z_{D,i}^{(1)}
\end{align*}
$$

where $h_{S,R}$ and $h_{S,D}$ denote the independent complex fading channel gain from $S$ to $R$ and $D$, modeled as $h_{S,R} \sim \text{CN}(0, \sigma_{S,R}^2)$ and $h_{S,D} \sim \text{CN}(0, \sigma_{S,D}^2)$ with $\sigma_{S,R}^2 = E[|h_{S,R}|^2]$ and $\sigma_{S,D}^2 = E[|h_{S,D}|^2]$ respectively. Without loss of generality, it is assumed that the noise terms $z_{R,i}^{(1)}$ and $z_{D,i}^{(1)}$ have equal
and are modeled as $z_{R,k}^{(i)} \sim \mathcal{CN}(0, N_0)$ and $z_{D,i}^{(i)} \sim \mathcal{CN}(0, N_0)$. Subscript 1 in $y_{R,k}^{(i)}$, $y_{D,i}^{(i)}$, $z_{R,k}^{(i)}$ and $z_{D,i}^{(i)}$ means the first time slot.

In the first transmission round, i.e., $k = 1$, the relay normalizes the received signal by a factor of $\beta^{(i)}$ (so that the average energy is unity) and retransmits the signal during the second time slot. During the $k$th, $k > 1$, transmission round, which happens if the destination fails to decode the packet in the previous transmission rounds, the relay combines the signals from the previous transmission rounds with the current signals using MRC before forwarding. Then, at the second time slot, the destination receives the signals given by

$$y_{D,2}^{(i)} = \begin{cases} \beta^{(i)}h_{R,D}y_{R,1}^{(i)} + z_{D,2}^{(i)}; & k = 1 \\ \beta^{(i)}h_{R,D} \sum_{n=1}^{k} h_{R,D}y_{R,n}^{(i)} + z_{D,2}^{(i)}; & k > 1 \end{cases}$$

(2)

where $h_{R,D}$ denotes the fading channel gain from R to D, $h_{R,D} \sim \mathcal{CN}(0, \sigma^2_{R,D})$ with $\sigma^2_{R,D} = E[h_{R,D}^2]$, $z_{D,2}^{(i)} \sim \mathcal{CN}(0, N_0)$ is additive noise at D, and

$$\beta^{(i)} = \begin{cases} \frac{1}{\sqrt{h_{R,D}^2 N_0}}; & k = 1 \\ \frac{1}{\sqrt{k^2 h_{R,D}^2 P_s + k h_{R,D}^2 N_0}}; & k > 1 \end{cases}$$

(3)

Hence, in the $k$th transmission round, the received signal after MRC combining at D can be expressed as

$$y_{D}^{(i)} = \begin{cases} \frac{h_{D}^{(i)}y_{D,1}^{(i)} + k \beta^{(i)}h_{R,D}h_{R,D}y_{R,1}^{(i)}}{N_0} + z_{D}^{(i)}; & k = 1 \\ \frac{h_{D}^{(i)}y_{D,1}^{(i)} + k \beta^{(i)}h_{R,D}h_{R,D}y_{R,1}^{(i)}}{N_0} + z_{D}^{(i)}; & k > 1 \end{cases}$$

(4)

Finally, all the signals transmitted in HARQ rounds are combined before decoding. The combined signal after $k$ transmission rounds is denoted by $y_{E}^{(i)}$, with ‘EQ’ meaning equal.

$$y_{E}^{(i)} = \sum_{n=1}^{k} y_{D}^{(i)}$$

(5)

### 3 Performance analysis

#### 3.1 BLER performance analysis

Define the instantaneous signal-to-noise ratio (SNR) at link S-R, R-D and S-D as $\gamma_{S,R} = \frac{h_{S,R}^2}{\sigma^2_{S,R}}$, $\gamma_{R,D} = \frac{h_{R,D}^2}{\sigma^2_{R,D}}$ and $\gamma_{S,D} = \frac{h_{S,D}^2}{\sigma^2_{S,D}} \odot \mathcal{F}$, respectively, with $\mathcal{F} = P_s / N_0$. Then, after MRC operations of all the receptions at D, the SNR of the overall received signal during the $k$th transmission round can be expressed as

$$\gamma_{D}^{(i)} = \gamma_{S,D} + \frac{k \gamma_{R,D} \gamma_{R,D}}{\gamma_{S,D} + \gamma_{R,D} + 1}; \quad k \in \{1, 2, ..., M\}$$

(6)

where $M$ is the maximum number of HARQ rounds, and $k \gamma_{S,D}$ is the SNR at R after it makes MRC of every transmission round of the same packet.

After $k$ transmission rounds, the overall SNR of the combined signal on MRC operation can be expressed as

$$\gamma_{E}^{(i)} = \sum_{n=1}^{k} \gamma_{D}^{(i)}$$

(7)

Note that errors at the destination only occur when all the previous (re) transmissions round are received in error. Hence, under the assumption of linear quadrature phase shift keying (QPSK) modulation, the AF relaying channel with HARQ after $k$ transmission rounds has end-to-end block error probability (BLER) conditioned on the instantaneous received SNRs, and it is given by Ref. [7]:

$$P[e | \gamma_{E}^{(i)}, \gamma_{E}^{(2)}] = \prod_{n=1}^{k} P[e | \gamma_{E}^{(n)}]$$

(8)

It is assumed that the terminated conventional codes with a finite uncoded block length $K$ and coded block length $N$ are considered in this article. Then, by using the weight enumerating function of the equivalent block code, the bound for the BLER conditioned on the instantaneous SNR of the $n$th transmission round can be calculated as

$$P[e | \gamma_{E}^{(n)}] \leq \frac{1}{\sum_{d=d_{n}}^{\infty} \sum_{w=1}^{K} a_{w,d} Q}\left(\sqrt{d_{n}^{(n)}}\right)$$

(9)

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp(-y^2/2)dy$ denotes the Gaussian $Q$-function, $d_{n}$ is the free distance of the code, $a_{w,d}$ is the multiplicity of code-words corresponding to input weight $w$ and output weight $d$. This expression employs a standard technique used for ideally interleaved fading channels and yields simple but very loose results. Then, the limit-before-average technique with the appropriate conditional BLER expressions is used to obtain tight bounds [8]. Hence, the BLER bound becomes

$$P[e | \gamma_{E}^{(n)}] \leq \min \left\{ 1, \sum_{d=d_{n}}^{\infty} \sum_{w=1}^{K} a_{w,d} Q\left(\sqrt{d_{n}^{(n)}}\right) \right\}$$

(10)

By inputting Eqs. (10) into (8), the bound for AF relaying channel with HARQ on the instantaneous received SNRs is given by

$$P[e | \gamma_{E}^{(1)}, \gamma_{E}^{(2)}] \leq \prod_{n=1}^{k} \min \left\{ 1, \sum_{d=d_{n}}^{\infty} \sum_{w=1}^{K} a_{w,d} Q\left(\sqrt{d_{n}^{(n)}}\right) \right\}$$

(11)
To obtain the average BLER, one must take the expected values of Eq. (11) over the distributions of fading coefficients:

\[ P_E^{(k)} = \frac{1}{\mathbb{E}} \mathcal{P} \{ x \} P(x \mid \bar{x}, \bar{y}_{SD}, \bar{y}_{SR}, \bar{y}_{R,D}, \bar{y}_{R,D} \} \]

(12)

where \( P_E^{(k)} \) means the average BLER after \( k \) transmission rounds and \( P(x) \) is the probability density function (pdf) of random variable \( x \). Under the Rayleigh fading channel, the fading coefficients of the links have an exponential distribution and their general form of pdf can be written as

\[ p(\gamma) = \frac{1}{\bar{\gamma}} \exp \left( -\frac{\gamma}{\bar{\gamma}} \right) \]  

(13)

Since the SNR values after several retransmissions are obtained, that is, Eq. (7), and they are all functions of \( \gamma_{SR} \), \( \gamma_{RD} \) and \( \gamma_{RD} \), by inputting Eqs. (7), (11) and (13) into Eq. (12), the bound of unconditional BER of arbitrary number of retransmission can be obtained:

\[ P_E^{(k)} \leq P_E^{(k)_{\text{upper}}} = \frac{1}{\mathbb{E}_{SD}} \mathcal{P}_{SD} \left\{ \sum_{n=1}^{k} \min \left\{ 1, \sum_{i=1}^{N} a_{n,i} Q \left( \sqrt{d_{n,i}} \right) \right\} \right\} \]

\[ \exp \left( -\frac{\gamma_{SD}}{\bar{\gamma}_{SD}} \frac{\gamma_{SR}}{\bar{\gamma}_{SR}} - \frac{\gamma_{RD}}{\bar{\gamma}_{RD}} \right) d\gamma_{SD} d\gamma_{SR} d\gamma_{RD} \]

(14)

where \( \bar{\gamma}_{SD} = \sigma_{SD}^{2}, \bar{\gamma}_{SR} = \sigma_{SR}^{2}, \bar{\gamma}_{RD} = \sigma_{RD}^{2} \).

3.2 Throughput analysis

The throughput is defined as

\[ \eta = \frac{N \hat{M}_a R (1 - P_e)}{LT} \]

(15)

where \( N \) is the number of symbols transmitted in one transmission round, \( \hat{M}_a \) is the number of bits carried by one modulation symbol, \( R \) is the code rate, \( P_e \) is the packet error rate which equals to BLER in this paper and \( L \cdot T \) is time consumed for \( L \) transmission round. Since \( N \) and \( T \) is constant for every transmission round, the normalize throughput is used instead:

\[ \eta = \frac{M \hat{M}_a R (1 - P_e)}{L} \]

(16)

Under the assumption that \( M \) is the maximum number of HARQ transmission rounds, the expected number of HARQ transmission rounds can be expressed by

\[ E[L] = \sum_{n=1}^{M} n P[L = n] \]

(17)

where \( L \) is the number of transmissions rounds, \( P[L = n] \) is the probability that the \( n \)th transmission round is happened and given by

\[ P[L = n] = \begin{cases} (1 - P_e)^{n-1} P_e, & n < M \\ \prod_{i=1}^{n-1} P_e, & n = M \end{cases} \]

(18)

Next, rewrite Eq. (17) by inputting Eq. (18), and the following can be obtained:

\[ E[L] = 1 + \sum_{n=1}^{M} \prod_{i=1}^{n-1} P_e \]

(19)

Finally, the normalized average throughput and its lower bound is given by

\[ \eta = \frac{M \hat{M}_a R (1 - P_e^{(k)_{\text{upper}}})}{E[L]} \geq \frac{M \hat{M}_a R (1 - P_e^{(k)_{\text{upper}}})}{1 + \sum_{i=1}^{M} \prod_{i=1}^{n-1} P_e} \]

(20)

4 Numerical results and simulations

The performances of HARQ transmission with AF relaying are evaluated by numerical analysis and discussed in this section. Also, the performances of single transmission are presented as reference. The family of rate-compatible punctured convolutional (RCPC) codes with generator polynomials \( G(171,133) \) is employed at rate \( R = 1/2 \), \( K = 200 \) and \( N = 400 \). The distance spectrum \( a_{n,d} \) is computed via computer enumeration for analysis. QPSK is employed in AF relaying transmission with HARQ. Since cooperation is adopted, the direct \( S \) to \( D \) link must be weaker than \( S \) to \( R \) and \( R \) to \( D \) link. Hence, the authors consider the representative scenarios in which the source node to relay node link has a better channel quality, and they set the corresponding average output SNRs \( \bar{\gamma}_{SR}, \bar{\gamma}_{RD}, \bar{\gamma}_{RD} \) in logarithmic-scale are \( \bar{\gamma} + 10 \text{ dB}, \bar{\gamma}, \bar{\gamma} + 2 \text{ dB} \), respectively.

As shown in Fig. 3, the BLER performances of AF relaying systems with and without HARQ transmission in two scenarios are obtained from the analysis and simulation, respectively. The average SNR is set to \( \bar{\gamma}_{SR}, \bar{\gamma}_{RD}, \bar{\gamma}_{RD} = \bar{\gamma} + 10 \text{ dB}, \bar{\gamma}, \bar{\gamma} + 2 \text{ dB} \) and the maximum number of transmission varies from one to four. The analytical results well match the simulation ones, which prove the effectiveness of the analysis method. These figures also show that the significant performance gain in term of SNR can be achieved by exploiting the HARQ diversity. With only one retransmission (i.e. \( M = 2 \)), the SNR gain is about 3 dB compared with the case of single-transmission (i.e. \( M = 1 \)) if the target BLER value is set to \( 10^{-3} \). Also, the gains for \( M = 3 \) and \( M = 4 \)
are given as 4.7 dB and 6 dB, respectively. In addition, the maximum number of (re)transmissions, i.e., $M$, in the practical systems is limited by the round trip time and delay requirements of the services.

Figs. 3 and 5 show the normalized throughput performances of HARQ transmission in AF relaying systems. Different scenarios such as $(7 + 10 \, \mathrm{dB}, 7, 7)$ and $(7 + 6 \, \mathrm{dB}, 7 + 2 \, \mathrm{dB}, 7)$ are considered. For clarity, only the results with $M = 1$ and $M = 4$ are given. It can be seen that HARQ transmission results in higher throughput than the single transmission under AF relaying channels in the low and medium SNR region. With the higher SNR, the smaller probability of retransmission happens and the gain due to HARQ diversity becomes less. These lower bounds also well match the simulation results.

To further study the gains of retransmissions, the performances of the systems with more number of (re)transmission are illustrated in Fig. 6. The throughput gain decreases with more number of (re)transmissions because the HARQ diversity becomes saturated with the large number of (re)transmissions. For example, in case of $M \geq 5$, the gain is even negligible rather than much delay. Thus, $M$ should be carefully designed such that both the quality of service and delay requirement can be satisfied.

5 Conclusions

This article analyzes the block error rate and throughput performance of AF relaying systems with HARQ transmission using weight enumerating functions. Numerical results with the proposed analytical method have been presented and their correctness has been validated by the simulation results. Also, it is shown that AF relaying system can benefit from HARQ transmission as a result of diversity gain, especially when the channel quality is not very good. Furthermore, the analytical
method can be extended to the systems with other HARQ protocols and other relaying schemes.

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References

2. 3GPP. TR25.913 V7.3.0. Requirements for evolved UTRA (E-UTRA) and evolved UTRAN (E-UTRAN). 2006

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