Modified precoding strategy over MIMO double-correlated channels

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Abstract

Double-correlated channels commonly occur due to insufficient scattering around both the transmitting and receiving antennas. In this article, the authors propose a linear precoding strategy with a modified system model over double-correlated channels. In this system model, an inner decorrelation strategy is added combined with precoding. First, a linear precoder that achieves minimum bit error rate (MBER) is proposed and analyzed based on the modified system model. The simulation results show that the bit error rate (BER) performance can be improved by decorrelation. Second, the influence of double correlation on multiple input and multiple output (MIMO) capacity is analyzed and simulated.

Keywords MIMO, precoding, decorrelation, capacity

1 Introduction

The multiple input and multiple output (MIMO) has become very popular as it can increase channel capacity [1]. It is known that optimal performance can be attained by power waterfilling across channel eigenvalues with a total transmitting power constraint [2]. In several publications [3–5], it has been shown that the performance of MIMO systems may be improved significantly by using some set of channel knowledge at the transmitter. In a number of applications, channel knowledge is also available at the transmitter. The channel can be estimated at the receiver and the feedback sent to the transmitter in the frequency division duplex (FDD) or it can be estimated at the transmitter during the receiving mode in the time division duplex (TDD). However, accurate instantaneous channel knowledge at the transmitter is difficult to acquire.

MIMO precoding [6–8] design has been an active research area. Many earlier designs focused on the perfect channel state information at the transmitter (CSIT) case often jointly optimizes both the precoder and decoder for various performance criteria based on the minimum square error (MSE) or signal to noise ratio (SNR). A more recent study has considered partial CSIT [9], but only the special cases of transmitting covariance CSIT and mean CSIT. These include precoders optimal for the channel ergodic capacity, given transmitting covariance CSIT or mean CSIT. Others are based on the error rate criterion with mean CSIT or transmitting covariance CSIT. The space-time block coding (STBC) encodes the input symbol stream across multiple antennas and across time to get transmitting diversity and coding gain. Typically, STBCs are often designed for i.i.d. Rayleigh fading channels, assuming there is no CSIT. The code design criterion assumes that the transmitting and receiving antennas are not correlated. However, such an assumption may not be true in practice. The transmitting antennas are correlated in the downlink because the base station antennas are normally high enough and have no local scatterers.

In this article, the authors study the design of a modified linear precoder for MIMO systems in spatially flat-fading channels. The objective is to minimize the BER and maximize the capacity. A recent study in Ref. [10] solved the problem with uncorrelated receive antennas. In this article, the authors consider double-correlated Rayleigh channels. In practice, double-correlated channels commonly occur because of insufficient scattering around both the transmitting and receiving antennas or around closely spaced antennas with respect to the wavelength of the signal.

2 Modified system model

Consider a MIMO system with $M_T$ transmitting and $M_R$ receiving antennas. The corresponding channel model for frequency-flat fading is given by
y = GHSFs + Gn

where \( s \) is an \( N \times M_t \) codeword matrix, \( F \) is an \( M_t \times N \) precoder matrix, \( G \) is an \( N \times M_t \) decoder matrix, \( H \) is an \( M_t \times M_t \) channel matrix, and i.i.d. is denoted by \( H \). The entries of the Rayleigh fading channel are assumed to be independent zero mean circularly symmetric complex Gaussian random variables. \( y \) is \( M_t \times N \) received signal matrix, and \( n \) is the \( M_t \times N \) noise matrix.

It is assumed that the noise is white across space and time. For simplicity of analysis, it is assumed that

\[
\begin{align*}
E[ss^H] &= \delta^2 I \\
E[nn^H] &= \delta^2 I = R_n \\
E[nn^H] &= 0
\end{align*}
\]

where \((\cdot)^H\) denotes the conjugate transpose. In the spatially correlated Rayleigh fading channel, the fading correlation \( R \) can be evaluated by

\[
R = E[HH^H]
\]

The correlation may be separated into two parts: the transmitter spatial correlation \( R_t \), and the receiver spatial correlation \( R_r \). Altogether the overall spatial correlation matrix \( R \) can be written as the Kronecker product

\[
R = R_t \otimes R_r
\]

Moreover, the channel matrix \( H \) can be written as

\[
H = R_t W R_r^{-1/2}
\]

where \( W \) is an i.i.d matrix with elements having a mean of 0 and a variance of 1. The square-roots \( R_t^{-1/2} \) and \( R_r^{-1/2} \) can be obtained by the eigenvalue decompositions of \( R_t \) and \( R_r \):

\[
\begin{align*}
R_t^{-1/2} &= U_t A_t^{-1/2} U_t^H \\
R_r^{-1/2} &= U_r A_r^{-1/2} U_r^H
\end{align*}
\]

where \( A_t \) and \( A_r \) are the diagonal matrices containing the eigenvalues \( \lambda_t \) and \( \lambda_r \) of \( R_t \) and \( R_r \), and \( U_t \), \( U_r \) are unitary matrices containing the corresponding eigenvectors.

Inserting Eq. (6) into Eq. (5), the MIMO channel model can be written as

\[
y = GU_s A_t^{-1/2} U_t^H W U_r A_r^{-1/2} U_r^H F s + Gn
\]

Define \( F = U_t \), \( G = U_r^H \), \( W' = U_r W U_t \), Eq. (7) can be rewritten as

\[
y = U_t^H U_r A_t^{-1/2} U_t^H W U_r A_r^{-1/2} U_r^H s + U_r^H n = A_t^{-1/2} W A_r^{-1/2} s + U_r^H n
\]

It can be seen that \( W' \) has the same statistics as \( W \). Define \( \hat{H} = A_t^{-1/2} W A_r^{-1/2} \) and it is found that the channel \( H \) was decorrelated.

3 Minimum BER design

In Ref. [11], exact closed-form expressions were derived for the symbol error rate in double-correlated Rayleigh environments with two antennas at either the transmitter or the receiver. However, they were too complicated to be used to design an optimal power allocation policy.

In this section, the authors apply all the general modulation formats that have an (symbol error probability rate) SER expression of the form

\[
P_b = aQ\left( \sqrt{2\gamma}\right)
\]

where \( Q() \) is the Gaussian Q-function, \( a \) and \( b \) are modulation-specific constants, and \( \gamma \) is the output SNR. Such modulation formats include binary phase-shift-keying (BPSK) \((a=1, b=1)\), and multiple phase-shift-keying (MPSK) \(a = 2, b = \sin^2(\pi/M)\).

For simplicity, it is assumed that input SNR \( \gamma = E_s/N_0 \) and BPSK modulation is used. The average BER can be calculated as

\[
\overline{P}_b = \frac{1}{M_t} \sum_{k=1}^{M_t} \mathcal{O}\left( \sqrt{2\gamma_k}\right)
\]

where \( \gamma_k \) is the decision-point SNR of the \( k \)-th signal stream, that is, the signal from the \( k \)-th transmit antenna, \( 1 \leq k \leq M_t \).

By denoting the power allocated to the \( k \)-th stream as \( P_k \), and defining \( P = \text{diag} \{ \sqrt{P_1}, \sqrt{P_2}, \ldots, \sqrt{P_{M_t}} \} \) as the power-allocation matrix, the total transmitting power is constrained as

\[
\text{tr}(PP^H) = \sum_{k=1}^{M_t} P_k = P_0
\]

The average BER for power allocation is given by

\[
\overline{P}_b = \frac{1}{M_t} \sum_{k=1}^{M_t} \mathcal{O}\left( \sqrt{2\gamma_k}P_k\right)
\]

The optimal power allocation policy is expressed mathematically as follows:

\[
P_k = \arg \min \frac{1}{M_t} \sum_{k=1}^{M_t} \mathcal{O}\left( \sqrt{2\gamma_k} P_k\right)
\]

s.t. \( \text{tr}(PP^H) = \sum_{k=1}^{M_t} P_k = P_0 \)

The Lagrangian is formulated as

\[
L = P_0 + \mu \left( \sum_{k=1}^{M_t} P_k - P_0 \right) = \frac{1}{M_t} \sum_{k=1}^{M_t} \mathcal{Q}\left( \sqrt{2\gamma_k} P_k\right) + \mu \left( \sum_{k=1}^{M_t} P_k - P_0 \right)
\]

where \( \mu \) is the Lagrange multiplier. The optimal solution to \( P \) satisfies the conditions:

\[
\frac{\partial L}{\partial P_k} = 0 \quad \text{for } k = 1, \ldots, M_t
\]

\[
\sum_{k=1}^{M_t} P_k = P_0
\]
From these equations, $p_i$ is obtained as

$$p_i = -\frac{2}{\ln M_i \mu} \ln \frac{\gamma_i}{\gamma}$$

(16)

where $\mu$ can be determined from the power constraint in Eq. (17):

$$\mu = \exp \left( \frac{\sum_{i=1}^{M} 2 \ln M_i}{\sum_{i=1}^{M} \gamma_i} \right)$$

(17)

4. Performance analysis and simulations

4.1 Influence of correlation on capacity

The capacity of the MIMO channel in the presence of the spatial fading correlation, with channel knowledge at the transmitter, follows from the simple substitution:

$$R = \log_2 \det \left( I_{M_R} + \frac{\gamma}{M_T} R_k^{\frac{1}{2}} HWR_t^{\frac{1}{2}} F F^H R_k^{\frac{1}{2}} H^H \right)$$

(18)

Following the modified system model Eq. (8), the capacity Eq. (18) can be rewritten as:

$$C = \log_2 \det \left( I_{M_t} + \frac{\gamma}{M_T} U^H R_k^{\frac{1}{2}} H W R_t^{\frac{1}{2}} U U^H \right)$$

$$R_k^{\frac{1}{2}} H W R_t^{\frac{1}{2}} U U^H = \log_2 \det \left( I_{M_t} + \frac{\gamma}{M_T} U^H R_k^{\frac{1}{2}} H W R_t^{\frac{1}{2}} U U^H \right)$$

(19)

This result implies that the decorrelation operation has no effect on the capacity.

4.2 Impact of imperfect CSIT

It has been assumed that the correlation matrix $R_t$ is perfectly known at the transmitter. In the case of estimation errors, the power allocation will be based on an erroneous transmitter correlation matrix $\hat{R}_t$. In general, both the eigenvectors and the eigenvalues of $\hat{R}_t$ will be different from those of the actual correlation matrix $R_t$, that is,

$$\hat{R}_t^{\frac{1}{2}} = \hat{U} \hat{A}_t^{\frac{1}{2}} \hat{U}^H$$

$$\hat{R}_k^{\frac{1}{2}} = \hat{U}_k \hat{A}_k^{\frac{1}{2}}$$

(20)

where $\hat{U}_i \neq U_i$ and $\hat{A}_t \neq A_t$. This has two effects: first, the transmitter will use a mismatched decorrelation matrix $\hat{U}_i$. If $\hat{U}_i \neq U_i$, the product $\hat{U}_i U_i$ does not yield the identity matrix. Second, the transmitter power allocation will be based on an erroneous eigenvalue of matrix $\hat{A}_t$. This means that a mismatched precoder will be used. Altogether, there will be an overall mismatch in the power allocation.

4.3 Simulations

A $(2 \times 2)$ STBC system with a transmitter correlation matrix $R_t$ and receiver correlation matrix $R_k$ is considered. Then, the performance improvement of this system using the precoder with decorrelation is depicted.

For the simulations, it is assumed that $R_t = \begin{bmatrix} 1 & 0 \\ R & 1 \end{bmatrix}$, $R_k = \begin{bmatrix} 1 & 0 \\ R_k & 1 \end{bmatrix}$

where $R_t$ and $R_k$ are correlation coefficients between the transmitters and receivers respectively.

Fig. 1 shows the average BER versus transmitter coefficient $R_t$ of BPSK in a double-correlated MIMO channel at SNR = 10 dB. It can be seen that the correlation coefficient is much larger and the BER is much higher. However, the BER can be improved if the authors use decorrelation. For example, when $R_t = 0.2$ and $R_k = 0.4$, the BER improves by 4.5% from $2.5 \times 10^{-2}$ to $3 \times 10^{-2}$ because of decorrelation.

Fig. 2 shows the capacity versus transmitter correlation coefficient $R_t$ in a double-correlated channel with $M_t = \begin{bmatrix} M_t & 2 \\ M_t & 2 \end{bmatrix}$ at SNR=10 dB. It can be seen that decorrelation has no effect on the capacity. Moreover, from Fig. 2 it can be seen that $R_t$ and $R_k$ have the same impact on the capacity of the MIMO channel. For example, when $R_t = R_k = 0.2$, the capacity is about 4.75 bit/(s·Hz); when $R_t = R_k = 0.4$, the capacity reduces to about 4.25 bit/(s·Hz). Thus, it has a loss of about 0.5 bit/(s·Hz) for the correlated channel ($R_t = R_k = 0.4$) compared with the uncorrelated channel ($R_t = R_k = 0$).
Fig. 2 Capacity of MIMO systems in spatially correlated channels: SNR = 10 dB

5 Conclusions

In this article, the authors proposed a modified precoding policy that adds a decorrelation stage for double-correlated MIMO systems. It was shown that this policy with decorrelation improves system performance compared with the strategy without decorrelation.

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References


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