Fairness based resource allocation for multiuser MISO-OFDMA systems with beamforming

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Abstract

Resource allocation problem in multiuser multiple input single output-orthogonal frequency division multiple access (MISO-OFDMA) systems with downlink beamforming for frequency selective fading channels is studied. The article aims at maximizing system throughput with the constraints of total power and bit error rate (BER) while supporting fairness among users. The downlink proportional fairness (PF) scheduling problem is reformulated as a maximization of the sum of logarithmic user data rate. From necessary conditions on optimality obtained analytically by Karush-Kuhn-Tucker (KKT) condition, an efficient user selection and resource allocation algorithm is proposed. The computer simulations reveal that the proposed algorithm achieves tradeoff between system throughput and fairness among users.

Keywords space-division multiple access (SDMA), multiuser diversity, proportional fairness, resource allocation

1 Introduction

The use of SDMA in the downlink of a multiuser multiple-input multiple-output (MU-MIMO) wireless communication network can provide a substantial gain in system throughput. By configuring multiple antennas at both base station (BS) and mobile station (MS), the capacity and link reliability of wireless communication are improved dramatically [1]. Spatial multiplexing (SM) gain increases spectral efficiency by opening up multiple spatial data pipes in the same frequency band of operation for no additional power expenditure [2]. Zero-forcing (ZF) technique or block diagonalization has been proposed for SDMA to remove co-channel interuser interference [3–4]. On the other hand, orthogonal frequency division multiplexing (OFDM) is an attractive modulation technique that splits the frequency selective broadband channel into a large number of flat fading subchannels and avoids serious intersymbol interference (ISI). The multiuser multiple input multiple output–orthogonal frequency division multiple access (MIMO-OFDMA) system with downlink beamforming can accommodate users in both frequency domain and spatial domain and provide finer granularity of resource allocation than pure time division multiple access or pure frequency division multiple access systems. Hence, it has great potential of capacity gain because of its multiuser diversity in space and frequency domain.

However, how spatial and frequency resources are efficiently allocated to active users such that both system throughput and a high degree of service fairness can be obtained is an important problem. The resource allocation in OFDMA system [5–6] and OFDMA/SDMA system [7–9] has been well studied. The work mainly focuses on maximizing system throughput subjected to total power constraint and BER requirement. Although the system throughput is maximized, the quality of service of users is not guaranteed because their data rates might be very low for some time intervals because of poor channel conditions. Therefore, balance between maximizing system throughput and ensuring that no user lacks resources should be found. The PF scheduling has been shown to achieve a good tradeoff between system throughput and fairness in the single antenna single-carrier system [10] and has been directly extended to multiuser MIMO system [11] and multiuser MIMO-OFDMA system [12]. The theoretic analysis of the fairness of the multiuser MISO-OFDMA system using ZF technique is...
considered in this article. We re-formulate the downlink PF scheduling problem as a maximization of the sum of logarithmic user date rate. The necessary condition on optimality is obtained analytically. The data rate of a user is not independent of the user set selected on the same subcarrier in (OFDMA)/SDMA systems. This make the problem more complicated and a suboptimal algorithm of low computational complexity is proposed.

The remainder of this article is organized as follows. Sect. 2 presents the system model and the formulation of the PF optimization problem. Sect. 3 introduces the optimality condition and the proposed algorithm. Sect. 4 presents the system model and the formulation of the PF optimization problem more complicated and a suboptimal algorithm of low computational complexity is proposed.

2 System model and problem formulation

2.1 System model

In a downlink multiuser MISO-OFDMA system, the BS is equipped with $N_i$ transmitting antennas, whereas each MS is equipped with one receiving antenna. The system has $N_c$ subcarriers and $K$ users, where $K \geq N_i$. The block diagram of the system is shown in Fig. 1. The fading channel between the BS and user $k$ on subcarrier $n$ is characterized by a $1 \times N_i$ vector $h_{k,n}$. It is assumed that the channel state information is perfectly known to the transmitter. The largest number of users that one subcarrier can simultaneously support is $N_i$.

There are $I$ possible combinations of users transmitting on the same subcarrier denoted as $A_i$, where $A_i \subset \{1,2,...,K\}$, $0 \leq |A_i| \leq N_c$. $|A|$ denotes the number of users in set $A_i$, and $I = \sum_{i=1}^{N_i} K$.

![Block diagram for a multiuser MISO-OFDMA system with downlink beamforming](image)

Assume that the user set selected on subcarrier $n$ is $A_i$, and the received signal of user $k$ $(k \in A_i)$ on subcarrier $n$ is

$$y_{k,n} = h_{k,n}\left(\sum_{j=1}^{K} p_{j,n} \omega_{j,n} b_{j,n}\right) + z_{k,n}$$

where $\omega_{k,n}$ is $N_i \times 1$ beamforming vector for user $k$ subject to $\omega_{k,n}^H \omega_{k,n} = 1$, $b_{k,n}$ is the transmitted data, $p_{k,n}$ is the allocated power, and $z_{k,n}$ is the complex Gaussian noise with variance $\sigma_n^2$. We use the ZF method [3-4] for spatial multiplexing to eliminate interuser interference on the same subcarrier, which means that $\omega_{k,n}$ is an orthogonal basis for the null space of a matrix with vector $h_{i,n}(j \in A_i, j \neq k)$ as its rows. Hence we have $h_{i,n}^H \omega_{k,n} = 0$, and

$$y_{k,n} = \sqrt{p_{k,n}} h_{k,n}^H \omega_{k,n} b_{k,n} + z_{k,n}$$

M-QAM modulation is applied with a BER requirement, and the data rate for user $k$ on the carrier $n$ can be expressed as

$$r_{k,n} = \log_2 \left(1 + p_{k,n} \gamma_{k,n} \right)$$

where $\gamma_{k,n} = \left|h_{k,n}^H \omega_{k,n}\right|^2 / \left(\sigma_n^2 \Gamma\right)$ is the equivalent signal to noise ratio and $\Gamma = -\ln \left(5B_\gamma / 1.5\right)$ [13], and $B_\gamma$ is the BER requirement of user $k$. The data rate of user $k$ can be written as

$$R_i = \sum_{n=1}^{N_c} \sum_{k=1}^{K} \rho_{k,n} \log_2 \left(1 + p_{k,n} \gamma_{k,n}\right)$$

where $\rho_{k,n}$ is defined as follows: $\rho_{k,n} = 1$ if user $k$ in the set $A_i$ and $A_i$ is selected on the subcarrier $n$; otherwise if $\rho_{k,n} = 0$.

2.2 Problem formulation

It is assumed that the equal power is distributed to all subcarriers. A proportional fairness optimization problem can be formulated as:

$$\max_{\rho_{k,n}, \gamma_{k,n}} \sum_{k=1}^{K} \ln R_i$$

s.t.

$$\sum_{k=1}^{K} p_{k,n} \leq \frac{P}{N_c}; \ \forall n, i$$

$$\sum_{k=1}^{K} \rho_{k,n} \leq N_i; \ \forall n, i$$

$$p_{k,n} \geq 0, \rho_{k,n} \in \{0,1\}; \ \forall n, i, k$$

where $k = 1,2,...,K_i$, $i = 1,2,...,I$, $n = 1,2,...,N_i$, and $P$ is the total transmitted power.

The optimization solution can be obtained by the method of exhaustive search of all possible user assignment sets on all...
subcarriers. The exhaustive search complexity is given by $P^N$, which is extremely complicated even for moderate $N$, and $K$. Another important problem that is worth careful consideration is that $\gamma_{i,j,s}$ is determined by the channel vector $h_{k,s}$ and the beamforming vector $\omega_{k,s}$ for user $k$, whereas $\omega_{k,s}$ depends on the user set $A_i$ selected for subcarrier $n$. Hence, the user rate $r_{i,s}$ is a function of the channel vector $h_{k,s}$, the user assignment set $A_i$, and the allocated power $p_{k,s}$. To precisely know the data rate $r_{i,s}$, the beamformer have to be calculated first, which is in general a computationally complex operation. This motivate us to find a more efficient solution with an acceptable performance.

3 Solution and proposed algorithm

In this section, we first discuss the optimality condition of Eqs. (5)–(8). From necessary conditions on optimality obtained analytically by KKT condition, a suboptimal algorithm of low computation complexity is proposed.

3.1 Optimality condition

**Proposition 1** To maximize the sum of logarithmic user data rate, subcarrier $n$ should be allocated to user set $A_i$ with user $k^*$, where

$$k^* = \arg \max_{k} \frac{r_{k,n}}{r_k}$$

(9)

and the allocated power of users in user set $A_i$ is given as

$$p_{k,j,s} = \left( \frac{1}{\lambda_{k,s} R_k \ln 2 - 1} \right)$$

(10)

where $(x)^+ = \max\{x,0\}$ and $\lambda_{k,s}$ is the water-filling level given by the solution to the equation $\sum_{k \in A_i} p_{k,j,s} \leq P_i / N_i$.

**Proof** Relating $\rho_{k,s}$ to a real number in $[0,1]$ to make the problem tractable. Use Lagrangian and we have

$$L = \sum_{k=1}^{N} \sum_{i=1}^{K} \rho_{k,i,n} \log_2 \left( 1 + P_{k,i,n} \gamma_{k,i,n} \right) -$$

$$\sum_{k=1}^{N} \sum_{i=1}^{K} \sum_{j=1}^{K} \sum_{k^*}^{K} \sum_{j=1}^{K} \beta_{k,j,s} p_{k,j,s} + \sum_{k=1}^{N} \sum_{i=1}^{K} \alpha_{k,s} \left( \sum_{k \in A_i} p_{k,j,s} - N_i \right) -$$

$$\sum_{k=1}^{N} \sum_{i=1}^{K} \sum_{j=1}^{K} \lambda_{k,s} \left( \sum_{k \in A_i} p_{k,j,s} - P_i / N_i \right) + \sum_{k=1}^{N} \sum_{i=1}^{K} \sum_{j=1}^{K} \omega_{k,j,s}$$

(11)

where $\lambda_{k,s}$, $\alpha_{k,s}$, $\beta_{k,j,s}$ and $\omega_{k,j,s}$ are nonnegative Lagrangian multipliers. According to KKT condition of optimality [14], we have

$$\frac{\partial L}{\partial p_{k,j,s}} = \frac{p_{k,j,s} \gamma_{k,j,s}}{R_k (1 + p_{k,j,s} \gamma_{k,j,s}) \ln 2} - \lambda_{k,s} + \beta_{k,j,s} = 0$$

(12)

$$\frac{\partial L}{\partial \rho_{k,j,s}} = \frac{r_{k,j,s}}{R_k} - \alpha_{k,s} + \omega_{k,j,s} = 0$$

(13)

$$\lambda_{k,s} \left( \sum_{k \in A_i} p_{k,j,s} - \frac{P_i}{N_i} \right) = 0$$

(14)

$$\alpha_{k,s} \left( \sum_{k \in A_i} p_{k,j,s} - N_i \right) = 0$$

(15)

$$\omega_{k,j,s} = 0$$

(16)

$$\beta_{k,j,s} = 0$$

(17)

From Eqs. (13) and (16), we can find that $\omega_{k,j,s}$ acts as a slack variable in Eq. (13), thus we can obtain

$$\frac{r_{k,j,s}}{R_k} - \alpha_{k,s} \leq 0$$

(18)

$$\rho_{k,j,s} \left( \frac{r_{k,j,s}}{R_k} - \alpha_{k,s} \right) = 0$$

(19)

According to Eqs. (18) and (19), there are two cases:

1) If user set $A_i$ is selected on subcarrier $n$ and $k \in A_i$, i.e., $\rho_{k,j,s} > 0$, then $\frac{r_{k,j,s}}{R_k} - \alpha_{k,s} = 0$.

2) If user set $A_i$ is not selected on subcarrier $n$ or $k \notin A_i$, i.e., $\rho_{k,j,s} = 0$, then $\frac{r_{k,j,s}}{R_k} - \alpha_{k,s} \leq 0$. This implies that subcarrier $n$ should be allocated to user set $A_i$ with user $k^*$ by Eq. (9).

Similarly, from Eqs. (12) and (17), we can obtain

$$\frac{p_{k,j,s} \gamma_{k,j,s}}{R_k (1 + p_{k,j,s} \gamma_{k,j,s}) \ln 2} - \lambda_{k,s} \leq 0$$

(20)

$$p_{k,j,s} \left( \frac{p_{k,j,s} \gamma_{k,j,s}}{R_k (1 + p_{k,j,s} \gamma_{k,j,s}) \ln 2} - \lambda_{k,s} \right) = 0$$

(21)

From Eqs. (20) and (21), if the user set $A_i$ is selected on subcarrier $n$ and $k \in A_i$, then $p_{k,j,s} > 0$, $\rho_{k,j,s} > 0$, and $\rho_{k,j,s} \gamma_{k,j,s} / \left( \frac{R_k (1 + p_{k,j,s} \gamma_{k,j,s}) \ln 2} - \lambda_{k,s} \right) = 0$. If the user set $A_i$ is not selected on subcarrier $n$ or $k \notin A_i$, then $p_{k,j,s} = 0$, $\rho_{k,j,s} > 0$, and $\rho_{k,j,s} \gamma_{k,j,s} / \left( \frac{R_k (1 + p_{k,j,s} \gamma_{k,j,s}) \ln 2} - \lambda_{k,s} \right) \leq 0$. It is possible that two user assignment sets have the same value in Eq. (9) (i.e., $r_{k,j,s} / R_k = r_{k,j,s} / R_k$, $k \in A_{h,l,j,s}$, $k \in A_{h,l,j,s}$, $i \neq j$). In this case, we will select the user set that has the larger sum data rate of users on the subcarrier $n$ and set $\rho_{k,j,s} = 1$. Therefore, the power allocated to user $k$ in user set $A_i$ on subcarrier $n$ is

$$p_{k,j,s} + \frac{1}{\gamma_{k,j,s}} = \frac{1}{\lambda_{k,s} R_k \ln 2}$$

(22)
Note that $p_{k,n} \geq 0$, which implies Eq. (10).

3.2 Proposed algorithm

At a decision epoch $t$, if the subcarrier $n$ is allocated to user $k$, then user $k$’s data rate is updated by

$$R_k(t) = R_k(t-1) + r_{k,n}$$

where $R_k(t-1)$ is user $k$’s data rate at $(t-1)$ epoch. The right hand side of Eq. (9) becomes

$$u_{k,n} = \frac{r_{k,n}}{R_k(t-1) + r_{k,n}}; \forall k \text{ and } n$$

which is an increasing function of $r_{k,n}$. This implies that the user with better channel condition takes higher priority in resource allocation. However, once user $k$ has been selected on subcarrier $n$, $R_k$ will be larger and the chance of selecting another subcarrier $n'$ ($n' \neq n$) will decrease at the next decision epoch $(t+1)$.

According to the proposition, the computational complexity reduces to $N^2$. As discussed above, the user data rate is affected by the user set selected on the same subcarrier. To precisely get $r_{k,n}$, the beamformers have to be computed first. We shall use a suboptimal method to further reduce search complexity. We first determine the number of users on each subcarrier according to capacity optimal rule.

The steps of the proposed algorithms are as follows:

1) Determine the number of users on each subcarrier, $(s_1, s_2, ..., s_N)$, by correlation-based algorithm. The choice of user number is determined by maximizing the sum of user data rate on the subcarrier. To reduce complexity, the correlation-based algorithm is used. A similar method has been proposed in Ref. [15]. Denote

$$\eta_{i,j} = \frac{H_{i,n}H_{j,n}^H}{\|H_{i,n}\|\|H_{j,n}\|}$$

as the correlation metric between user $i$ and user $j$ on subcarrier $n$, where $\| \|$ represents Euclidean norm. The larger $\eta_{i,j}$ is, the higher the correlation of two channel vectors is. On each subcarrier, the user with the largest channel norm is selected to exploit multiuser diversity gain. The second user is selected from the remaining users according to Eq. (25), i.e., the user of the smallest correlations coefficient with the first selected user is selected.

2) Selection of the proper user set belongs to $\Phi_{s_i}$ according to the PF policy in Eqs. (9) and (24) and the allocation of power to users through Eq. (10). Here, $\Phi_{s_i}$ denotes the set of the users sets with the same cardinality on the subcarrier $n$, i.e., $\Phi_{s_i} = |A_i|: |A_i| = s_i$, $A_i \subset \{1,2, ..., K\}$, $s_i = 1,2, ..., N_i$, $n = 1,2, ..., N$. By doing so, the complexity is reduced to

$$\sum_{n=1}^N \|r_{k,n}\|.$$
as a subband. The multipath fading channel model is the tapped delay-line spatial channel model (SCM) [16]. The carrier frequency is 2 GHz. Assume that each MS is equipped with 1 antenna and the BS is equipped with 4 antennas. Mobile users are assumed to have heterogeneous path losses of variance 2 dB. Let the acceptable BER be $10^{-6}$. The available modulation orders are constrained to \{0, 2, 4, 16, 64\}, where 0 means that no data is transmitted. Each MS moves randomly with a velocity of 3 km/h.

Fig. 2 shows the sum data rate achieved by the PS-GCM in Ref. [9], the proposed algorithm, RR with water-filling power allocation and RR with equal power allocation. As we can see, the total system capacity of the proposed algorithm is slightly degraded compared with PS-GCM algorithm and significantly enhanced over RR algorithm either with equal power allocation or with water-filling power allocation. When SNR is 6 dB, there is only a 4% reduction in the sum data rate of the proposed algorithm compared with PS-GCM. On the other hand, the proposed algorithm has a capacity improvement over RR with water-filling power allocation by 300%.

Both Figs. 3 and 4 illustrate the fairness comparison when SNR is 6 dB. The users are labeled by their path losses; users closer to the BS have smaller IDs. In PS-GCM, the users with smaller IDs have higher data rates as they have good channel conditions and occupy more subchannels. However, this scheme makes the data rates of the users with larger IDs smaller. The data rate of the user with the largest ID is only 27% of that of the user with the smallest ID. Comparatively, the proposed algorithm has approximately equal fairness among all users. The users with larger IDs have more chances to occupy the subchannels compared with PS-GCM. The data rate of the user with the largest ID is 70% of that of the user with the smallest ID. Both RR with equal power allocation and RR with water-filling power allocation achieve good fairness among users. This is because all users have equal chance of occupying the subchannels in RR algorithms. But the system throughput of both RR algorithms is much less than that of the proposed algorithm.

Therefore, it can be said that the proposed algorithm provides a good tradeoff for system throughput and fairness among users.

5 Conclusions

This study mainly investigates the PF problem for multiuser MISO-OFDMA systems with downlink beamforming. We formulate the PF problem and get the necessary condition on optimality analytically, then propose an efficient user selection and resource allocation algorithm with low complexity. The computer simulations show that the proposed algorithm attains a good balance between the system throughput and the fairness among users.

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