Teleportation of an unknown two-particle entangled state via an asymmetric three-particle entanglement state

Abstract In this article, a protocol for the teleportation of an unknown two-particle entanglement is proposed. The feature of the present protocol is that we utilize an asymmetric three-particle entangled state as the quantum channel. The optimal discrimination between two nonorthogonal quantum states is adopted. It is shown that an unknown two-particle entangled state can be probabilistically teleported from the sender to the remote receiver on condition that the co-sender successfully collaborates. The fidelity in this protocol is one. In addition, the probability of the successful teleportation is calculated and all kinds of transformations performed by the sender and the receiver are provided in detail.

Keywords teleportation, two-particle entangled state, asymmetric three-particle entangled state, generalized measurement

1 Introduction

In the burgeoning field of quantum information processing, scientists have made dramatic progress with quantum teleportation in theory and experiment [1−4]. Quantum teleportation, which has been considered to be one of the striking nontrivial techniques, allows for the transmission of quantum information to a distant location via a quantum channel with the help of local operation and classical communications.

Since the seminal work of Bennett et al. [1] in 1993, quantum teleportation has been paid considerable attention owing to its important application in quantum communication and quantum calculation. At present, various kinds of quantum teleportation have been proposed [5−12]. As far as pure three-qubit states are concerned, three entangled qubits can be classified into two inequivalent classes, such as greenberger-horne-zeilinger (GHZ) type and \( W \) type, which are all symmetric three-particle states, under stochastic local operations and classical communication transformations. It has been shown that when the three-particle GHZ state is shared as quantum channel [5], the fidelity of teleportation for a single particle state is \( F_{\text{GHZ}} = 2/3 + (\sin 2\theta)/3 \) and when the three-particle \( W \) state is utilized [6], the fidelity is \( F_{\text{W}} = 7/9 \). At the same time, the three-qubit states can be divided into five types [14], where asymmetric entangled states appear. Recently, based on the three-qubit states classification by unitary transformation, Bae et al. [15] considered three-party quantum teleportation with asymmetric states, and found that when the three parties take the state \( |\psi\rangle = (|000\rangle + |011\rangle + |111\rangle)/\sqrt{3} \) as quantum channel, there was an optimal scenario, which produces the fidelity \( F_\psi = 8/9 \). The fidelity between the state that the first party wishes to send and the state that the final party receives not only quantifies how successfully the parties accomplish quantum teleportation, but also implies the robustness of the quantum state. In other words, the asymmetric state is more useful sometimes as the quantum channel.

Quantum entanglement plays a central role for some applications in quantum information processing. Due attention has been paid to generating entanglement, nevertheless, quantum entanglement has to be transported among spatially separated spots in some applications, for example quantum computation [16]. Hence, it is most advantageous to transmit the entanglement initially imposed on an unknown multipartite state to a multipartite state at a remote place in quantum information. Owing to Schmidt decomposition [17], any partially entangled state of a bipartite system may be expressed as a linear combination of biorthogonal product states, i.e.,

\[
|\phi\rangle_{12} = x|00\rangle_{12} + y|11\rangle_{12}
\]

where \( x \) and \( y \) are unknown non-negative real numbers satisfying \( x^2 + y^2 = 1 \). Two-party quantum teleportation of the above two-particle entangled state via symmetric three-particle GHZ class state [9] and symmetric three-particle \( W \) class state [11] is investigated, respectively. The purpose of the present article is to teleport the above two-particle entangled state using an asymmetric three-particle entangled
state among the three-party.

In this article, we present a scheme for teleporting an unknown two-particle entanglement among asymmetric three parties. Asymmetric teleportation [15] for a single particle state will carry more information than symmetrical ones in the three-party quantum teleportation. In the present scheme, the quantum channel is an asymmetric three-particle entangled state. Furthermore, a local generalized measurement described by a positive operator-valued measure (POVM), which may allow the extraction of more information than the usual von neumann-type projective measurement, is adopted by the present article in the course of teleportation. In addition, the probability to obtain the successful teleportation is calculated. All kinds of transformations that Alice and Bob require to perform are provided in detail. It is shown that the asymmetric teleportation for an unknown two-particle entangled state can be completed with a certain probability on condition that the co-sender successfully collaborates through local operation and classical communication.

2 Teleportation of a two-particle entangled state

Suppose that an unknown two-particle entangled state represented by Eq. (1) requires to be transmitted between two parties, traditionally called Alice (sender) and Bob (receiver), who are spatially separated, with the help of the third party Cliff (co-sender).

The following asymmetric state, which is a general state corresponding to the state used by Bae et al in Ref. [15], is utilized as the quantum channel.

\[
|\Phi_{+}\rangle_{12345} = a|000\rangle_{1235} + b|011\rangle_{1235} + c|111\rangle_{1235}
\]

(2)

Here, the coefficients \(a\), \(b\), and \(c\) are real numbers and satisfy \(a^2 + b^2 + c^2 = 1\), \(a > b > c > 0\). Alice is in possession of particles 1, 2, 4, particle 3 in Cliff’s possession, and particle 5 in Bob’s possession. Therefore, the state of the whole system composed of particles 1, 2, and the quantum channel at the beginning of the protocol is simply

\[
|\Phi_{+}\rangle_{12} \otimes |\Phi_{+}\rangle_{345}
\]

(3)

To teleport the two-particle entangled state \(|\Phi_{+}\rangle_{12}\), firstly, Alice performs joint measurement on particles 1 and 4 in the Bell basis. As a result, the state of particles 2, 3, and 5 collapses into one of the following unnormalized states at random (4 kinds in all).

\[
\langle \Phi^{+}_{14} \rangle_{12345} = \frac{1}{\sqrt{2}} (a|x00\rangle_{235} + b|101\rangle_{235} + c|111\rangle_{235})
\]

(4)

\[
\langle \Psi^{+}_{14} \rangle_{12345} = \frac{1}{\sqrt{2}} (a|x01\rangle_{235} + b|011\rangle_{235} + c|000\rangle_{235})
\]

(5)

The four Bell states of particles 1 and 4 can be expressed as

\[
|\Phi^{+}_{14}\rangle = (|00\rangle_{14} \pm |11\rangle_{14})/\sqrt{2}, \quad |\Psi^{+}_{14}\rangle = (|01\rangle_{14} \pm |10\rangle_{14})/\sqrt{2}
\]

After that, if Cliff would like to help Alice and Bob to complete the teleportation, he should perform a von neumann measurement on particle 3. The state of particles 2 and 5 conditioned on Cliff’s measurement result

\[
|0\rangle_{3} \text{ collapses into:}
\]

\[
|0\rangle_{12345} (\langle \Phi^{+}_{14} \rangle_{12345} = \frac{1}{\sqrt{2}} (ax|x00\rangle_{235} + by|111\rangle_{235})
\]

(6)

\[
|0\rangle_{12345} (\langle \Psi^{+}_{14} \rangle_{12345} = \frac{1}{\sqrt{2}} (bx|x01\rangle_{235} + ay|100\rangle_{235})
\]

(7)

Otherwise, the state will be

\[
|1\rangle_{12345} (\langle \Phi^{+}_{14} \rangle_{12345} = \frac{1}{\sqrt{2}} (cy|x01\rangle_{235})
\]

(8)

\[
|1\rangle_{12345} (\langle \Psi^{+}_{14} \rangle_{12345} = \frac{1}{\sqrt{2}} (cx|x01\rangle_{235})
\]

(9)

Obviously, if the result of the measurement on particle 3 is \(|1\rangle_{3}\), the teleportation fails. On the contrary, if Cliff gains the result \(|0\rangle_{3}\), the teleportation will continue.

Bob introduces an additional qubit \(A\) with the state \(|0\rangle_{A}\), because the entanglement teleportation transfers not only the amount of entanglement but also the entanglement structure. At the final stage of the teleportation, the original state of qubits 1 and 2 initially possessed by Alice will be restored on qubits 5 and \(A\) owned by Bob.

To begin with, Bob performs two-particle entangling transformation on particles 5 and \(A\), which should have a nonzero entangling power. In the teleportation of a two-particle entangled state [10], the controlled-NOT (CONT) operation has been utilized. Furthermore, the quantum CONT gate has been demonstrated successfully in Ref. [18]. In this article, we also take this entangling technique, that is, the CNOT operation, to realize our scheme of teleportation instead of generic two-particle entangling transformation to implement easily.

Bob performs the CNOT operation with particle 5 as the control qubit and particle \(A\) as the target qubit. The following states can be obtained:

\[
|\Phi^{\circ}_{25}\rangle = \frac{1}{\sqrt{2}} (ax|x00\rangle_{254} \pm by|111\rangle_{254})
\]

(10)

\[
|\Phi^{\circ}_{25}\rangle = \frac{1}{\sqrt{2}} (bx|x01\rangle_{254} \pm ay|100\rangle_{254})
\]

(11)

Without loss of generality, suppose that the state of particles 2, 5, and \(A\) conditioned on Alice’s measurement result \(|\Phi^{+}_{14}\rangle_{14}\) is \(|\Phi^{\circ}_{25}\rangle_{254}\). We note that the state can also be expressed as:

\[
|\Phi^{\circ}_{25}\rangle = \frac{\sqrt{a^2 + b^2}}{2\sqrt{2}} [ (x|x00\rangle_{54} \pm y|11\rangle_{54}) |u_2\rangle + (x|x00\rangle_{54} \mp y|11\rangle_{54}) |v_2\rangle ]
\]

(12)
where \( |u\rangle_2 = \frac{a}{\sqrt{a^2 + b^2}} |0\rangle_2 + \frac{b}{\sqrt{a^2 + b^2}} |1\rangle_2 \), \( |v\rangle_2 = \frac{a}{\sqrt{a^2 + b^2}} |0\rangle_2 - \frac{b}{\sqrt{a^2 + b^2}} |1\rangle_2 \).

Of course, all that we have done is re-written the state in a particular way; nothing has changed physically.

Looking closely at Eq. (12) we notice that the relative states of particles 5 and \( A \) have a very similar relation to the initial unknown state \( |\phi\rangle_{12} \). Therefore, Alice’s task is to perform a measurement on particle 2 to distinguish the two nonorthogonal states \( |u\rangle_2 \) and \( |v\rangle_2 \). It is known that if one wants to be able to distinguish conclusively between two nonorthogonal photon polarization states at least some of the time, it is useful to consider a positive operator valued measure.

The following positive operators can be verified straightforwardly to form a POVM. If Alice applies the POVM on particle 2, then, after the measurement, particles 5 and \( A \) owned by Bob would have jumped instantaneously into one of the states \( x|0\rangle_{5,4} + y|1\rangle_{5,4} \) or \( x|0\rangle_{5,4} - y|1\rangle_{5,4} \) with equal probability. The optimal probability of obtaining a conclusive result from such a generalized measurement POVM is \( 2b^2/a^2 + b^2 \).

\[
\begin{align*}
E_1 &= \frac{1}{2a^2} \begin{pmatrix} b^2 & ab \\ ab & a^2 \end{pmatrix} \\
E_2 &= \frac{1}{2a^2} \begin{pmatrix} b^2 & -ab \\ -ab & a^2 \end{pmatrix} \\
E_3 &= I - E_1 - E_2 = \begin{pmatrix} 1 - \frac{b^2}{a^2} & 0 \\ 0 & 0 \end{pmatrix}
\end{align*}
\]

To make Bob know the state acquired with certainty, via the classical channel, Alice sends a message to Bob instructing him about the outcomes of her measurement.

With these classical bits come from Alice in hand, Bob can confirm the quantum state composed of particles 5 and \( A \). For the outcome \( x|0\rangle_{5,4} + y|1\rangle_{5,4} \), Bob must not do anything further. In the other case, for the outcome \( x|0\rangle_{5,4} - y|1\rangle_{5,4} \), Bob must perform unitary transformation \( (\sigma_y)_A I_5 \) to convert his particle 5 and \( A \) into a replica of Alice’s original state \( |\phi\rangle_{12} \). Meanwhile, no trace of identity of the unknown state remains in Alice’s region, as is required in accordance with the no-cloning theorem.

If the state of particles 2, 5, and \( A \) conditioned on Alice’s measurement result \( |\psi\rangle_{5,4} \) is \( |\phi_{25,4}\rangle \), Alice must perform unitary transformation \( (\sigma_y)_2 \) on particle 2. Then, the following states can be obtained:

\[
|\phi_{25,4}\rangle = \sqrt{a^2 + b^2} \left( (x|1\rangle_{5,4} + y|0\rangle_{5,4})|u\rangle_2 - (x|1\rangle_{5,4} - y|0\rangle_{5,4})|v\rangle_2 \right)
\]

Similarly, Alice performs an optimal POVM expressed by Eq. (13) to conclusively distinguish the nonorthogonal states between \( |u\rangle_2 \) and \( |v\rangle_2 \). After receiving the record of Alice’s measurement outcome, Bob performs appropriate unitary transformation to obtain the state that Alice was trying to transmit. All possible transformations performed by Alice and Bob are provided in Table 1, where we observe that under the Cliff’s successful help, Bob’s necessary unitary operation depends only on Alice’s measurement result. For convenience, Alice’s Bell-state measurements on particles 1 and 4 are denoted as \( BM_{14} \).

### Table 1

<table>
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<tr>
<th>Bob’s unitary operation</th>
<th>States of particles 5 and ( A )</th>
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<td>( (\sigma_z)_A \otimes I_4 )</td>
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By synthesizing all conditions (four kinds), we can see that teleportation is successfully realized with the probability of \((a^2 + b^2)/8 \times 2 \times (2b^2)/(a^2 + b^2) = 2b^2/a^2 \). If the relationship among coefficients \( a \), \( b \), and \( c \) is \( a = b = c = 1/\sqrt{3} \), the total probability of successful teleportation is \( 2/3 \). The fidelity for the present protocol can reach one.

### 3 Discussion and conclusions

Generally, it is not possible to deterministically and exactly recover the unknown state using the partially entangled state as a quantum channel [4–9, 11, 12]. In this case, one can only obtain the approximate teleportation deterministically (with imperfect fidelity but unit success probability) or the exact teleportation probabilistically (with unit fidelity but certain probability of failure). In this article, we considered the exact teleportation probabilistically for an unknown two-particle entangled state via an asymmetric three-particle entangled state. Certainly, one can also study the approximate teleportation deterministically for the two-particle entangled state via the same quantum channel in the future.
In conclusion, the central theme of this article is to teleport an unknown two-particle entangled state via an asymmetric state initially shared among three parties. In this protocol, Alice must perform a Bell measurement and a local generalized measurement described by POVM, and Bob must introduce an additional particle and adopt an appropriate unitary transformation. It is shown that an unknown two-particle entangled state can be probabilistically teleported between two parties on condition that the third party successfully collaborates.

Acknowledgements This work is supported by the Hi-Tech Research and Development Program of China (2006AA01Z419), the National Natural Science Foundation of China (90604023), the National Laboratory for Modern Communications Science Foundation of China (9140C110101601), the Natural Science Foundation of Beijing (4072020), and the ISN Open Foundation.

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